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CONCORDIA UNIVERSITY
Faculty of Engineering and Computer Science
Department of Mechanical and Industrial Engineering
MECHANICAL ENGINEERING DESIGN - MECH 441/2
Fall 2002

Instructors: V.N.Latinovic
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ASSIGNMENT 1

Due date: Monday, September 30, 2002

Solve the following problems from the textbook and a reference:

1. Problem 4-1 for values in rows ***(c) and (o)*** Table P4-1, stress in [MPa], (page 239).
2. Problem 4-8, (page 240).
3. Problem 4-19, (page 242).
4. Problem 4-33 for values in row ***(b)***, (page 245-246).
5. For the von Mises criterion in terms of principal stresses $\sigma_1^2 - \sigma_1\sigma_3 + \sigma_3^2 = S_y^2$ and equations for the two principal stresses given below obtain an equivalent equation in terms of *components* σ_x, σ_y and τ_{xy}.

$$\sigma_1, \sigma_3 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

6. For the common case of a single normal and a single shear stress, as in a shaft under bending and torsion, to what does the equation of Question 5 reduce? Derive a similar equation based on the maximum shear-stress theory of failure, and compare.

The problems are equally valued at 5 marks each for a total of 30 marks.

Assignment 1

MECH441/2 — Fall 2002

Problem 1 (P. 4.1) (c)

Given: $\sigma_x = 500\ MPa$

$\sigma_y = -500\ MPa$

$\tau_{xy} = 1000\ MPa$

$$\sigma_{1,3} = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2} = \frac{500+(-500)}{2} \pm \sqrt{\left(\frac{500-(-500)}{2}\right)^2+1000^2}$$

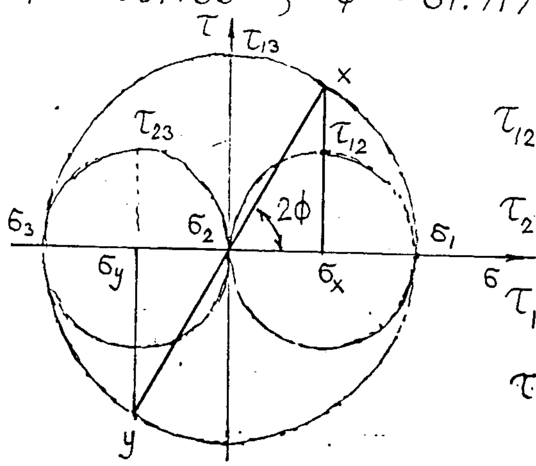
$$\sigma_1 = 0 + \sqrt{250000+1000000} = 1118.034\ MPa$$

$$\sigma_3 = 0 - 1118.034 = -1118.034\ MPa$$

$$\sigma_2 = 0$$

$$\tan 2\phi = \frac{2\tau_{xy}}{\sigma_x-\sigma_y} = \frac{2(1000)}{500-(-500)} = \frac{2000}{1000} = 2$$

$$2\phi = 63.435^\circ\ ;\quad \phi = 31.717^\circ$$

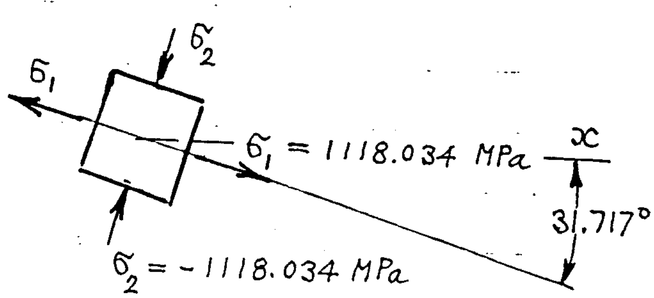


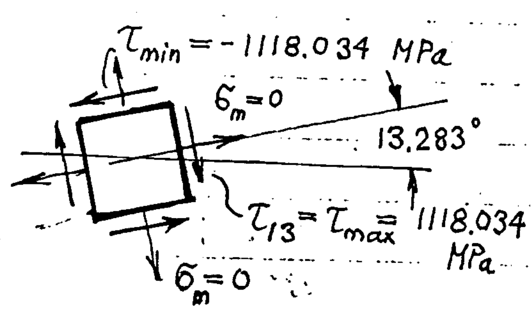
$$\tau_{12} = \frac{1}{2}|\sigma_1-\sigma_2| = \frac{1}{2}(1118.034) = 559.017\ MPa$$

$$\tau_{23} = \frac{1}{2}|\sigma_2-\sigma_3| = \frac{1}{2}|-1118.034| = 559.017\ MPa$$

$$\tau_{13} = \frac{1}{2}|\sigma_1-\sigma_3| = \frac{1}{2}|1118.034-(-1118.034)|$$

$$\tau_{13} = \tau_{max} = 1118.034\ MPa$$





P. 4.1 (0)

Given: $\sigma_x = 1000$ MPa $\quad \tau_{xy} = 500$ MPa
$\sigma_y = 1000$ MPa $\quad \tau_{yz} = 500$ MPa
$\sigma_z = 1000$ MPa $\quad \tau_{zx} = 0$ MPa

Cubic equation: $\sigma^3 - C_2\sigma^2 - C_1\sigma - C_0 = 0$

$$C_2 = \sigma_x + \sigma_y + \sigma_z = 1000 + 1000 + 1000 = 3000$$

$$C_1 = \tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2 - \sigma_x\sigma_y - \sigma_y\sigma_z - \sigma_z\sigma_x$$
$$= 500^2 + 500^2 + 0^2 - 1000(1000) - 1000(1000) - 1000(1000)$$
$$= 2500000$$

$$C_0 = \sigma_x\sigma_y\sigma_z + 2\tau_{xy}\tau_{yz}\tau_{zx} - \sigma_x\tau_{yz}^2 - \sigma_y\tau_{zx}^2 - \sigma_z\tau_{xy}^2$$
$$= 10^9 + 0.00 - 1000(500)^2 - 1000(0)^2 - 1000(500)^2$$
$$= 10^9 - 2(1000)500^2 = 1000(10^6) - 500(10)^6 = 500(10)^6$$

$$\sigma^3 - 3000\sigma^2 + 2500000\sigma - 500000000 = 0$$

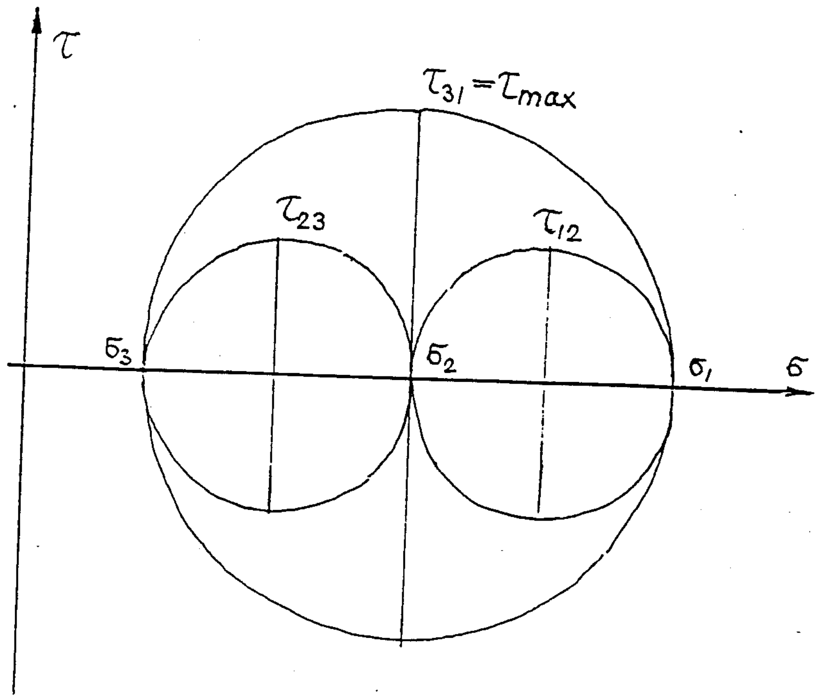
Try $\sigma = 1000$

$$1000^3 - 3000(1000)^2 + 2500000(1000) - 500000000 = 0$$

$$\sigma^3 - 3000\sigma^2 + 2500000\sigma - 500000000 \div (\sigma - 1000)$$
$$-\sigma^3 \pm 1000\sigma^2$$
$$0 - 2000\sigma^2 + 2500000\sigma - 500000000$$
$$\pm 2000\sigma^2 \mp 2000000\sigma$$
$$0 + 500000\sigma - 500000000$$
$$\mp 500000\sigma \pm 500000000$$
$$0 \qquad 0 \qquad = \sigma^2 - 2000\sigma + 500000 = 0$$

$$\sigma = 1000 \pm \sqrt{1000^2 - 500000} = 1000 \pm 707.1068 \text{ MPa}$$

$\sigma_1 = 1707.1068$ MPa $\quad \tau_{12} = \frac{1}{2}(707.1068) = 353.5534$ MPa
$\sigma_2 = 1000.00$ MPa $\quad \tau_{23} = \frac{1}{2}(707.1068) = 353.5534$ MPa
$\sigma_3 = 292.8932$ MPa $\quad \tau_{31} = \tau_{max} = \frac{1}{2}(1414.2136) = 707.1068$ MPa



Problem 2 (P.4.8)

Given
$\rho = 984\ kg/m^3$ $\quad d_o = 220\ mm$
$D_o = 1.50\ m$ $\quad E = 207(10^9)\ Pa$
$D_i = 0.22\ m$ $\quad \delta = 3\ mm$
$L = 3.23\ m$

1. Weight of the paper roll

$$W_r = \frac{\pi}{4}(D_o^2 - D_i^2)L\rho g = \frac{\pi}{4}(1.5^2 - 0.22^2)3.23(984)9.81$$

$$W_r = 53913.18\ N$$

2. Distributed load: $w = \frac{W_r}{L} = \frac{53913.18}{3.23} = 16691.39\ N/m$

3. Maximum deflection at the midspan

$$y_{max} = \frac{5wL^4}{384EI} = \delta\ ;\ I = \frac{5wL^4}{384E\delta}\ ;\ \delta = 0.003\ m$$

$$I = \frac{5(16691.39)3.23^4}{384(207)10^9(0.003)} = 38.0934(10^{-6})\ m^4$$

$$I = \frac{\pi}{64}d_o^4(1-\psi^4) = \frac{\pi}{64}0.22^4(1-\psi^4) = 1.14990(10^{-4})(1-\psi^4)$$

$1-\psi^4 = 0.33127576$; $\psi = 0.904298$; $d_i = \psi d_o = 0.904298(220)$

$d_i = 198.94\ mm = 198\ mm$ rounded off (safe). ANS

Problem 3 (P.4.19)

From problem 4.18 $\sigma = 30$ kpsi, $N = 10$ rods, $d_r = 0.75$ in

$l = 30$ ft; $F = ?$

$$\Delta l = \frac{F_1 l}{AE} = \sigma \frac{l}{E} = 30000 \frac{30(12)}{30(10^6)} = 0.36 \text{ in}$$

1. Load $F_1 = \frac{\Delta l}{l} AE = \frac{0.36}{30(12)} \frac{\pi d_r^2}{4} E = \frac{0.36}{30(12)} \frac{\pi (0.75)^2}{4} 30(10^6)$

$F_1 = 13253.594$ lb; $F = N F_1 = 10(13253.594)$

$F = 132535.94$ lb load in the clevis

2. Clevis size

$t = 0.8$ in; $\sigma_{all} = 40$ kpsi; $\tau_{all} = 20$ kpsi

Clevis pin diameter

$$\tau_{pin} = \frac{F}{2 A_{pin}} = \frac{F}{2 \frac{\pi d_{pin}^2}{4}} = \frac{2F}{\pi d_{pin}^2} \leq \tau_{all}$$

$$d_{pin} = \sqrt{\frac{2F}{\pi \tau_{all}}} = \sqrt{\frac{2(132535.94}{\pi (20000)}} = 2.054 \text{ in; use } 2.10 \text{ in}$$

3. Bearing stress in the clevis:

$$\sigma_b = \frac{F}{A_b} = \frac{F}{2 d_{pin} t} = \frac{132535.94}{2(2.1)0.8} = 39445.22 \text{ psi} < \sigma_{all}$$

4. Tearout shear stress:

$$\tau = \frac{F}{4t\sqrt{R^2 - (0.5 d_{pin})^2}} \; ; \; R = \sqrt{\left(\frac{F}{4t\,\tau_{all}}\right)^2 + [0.5 d_{pin}^2]^2}$$

$$R = \sqrt{\left(\frac{132535.94}{4(0.8)20000}\right)^2 + [0.5(2.1)]^2} = 2.322 \text{ in}$$

Rounded off to $R = 2.35$ in

5. Tension in the clevis weakest section:

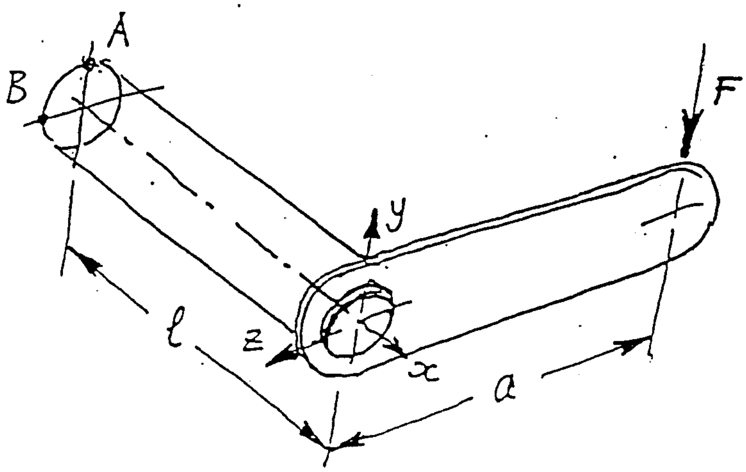
$$\sigma = \frac{F}{2(2R - d_{pin})} = \frac{132535.94}{2[2(2.35) - 2.1]} = 25487.68 < \sigma_{all}$$

(4) Problem 4-33

DATA for (b)

$\ell = 70\ mm$; $a = 200\ mm$; $t = 6\ mm$; $h = 80\ mm$;

$F = 85\ N$; $OD = 20\ mm$; $ID = 6\ mm$; Steel



$$\psi = \frac{d_i}{D_o} = \frac{6}{20} = 0{,}3$$

LOADS AT A & B :

Torque : $T = Fa = 85\,(0{,}20) = 17.0\ Nm$

Bending moment : $M = F\ell = 85\,(0{,}07) = 5.95\ Nm$

Shear force : $V = F = 85\ N$

Stresses at A :

$$\sigma_x = \frac{32M}{\pi D^3(1-\psi^4)} = \frac{32\,(5.95)}{\pi\,(0{,}020^3)(1-0{,}3^4)} = 7.6376\ MPa$$

$$\tau_{xz} = \frac{16T}{\pi D^3(1-\psi^4)} = \frac{16\,(17.0)}{\pi\,(0{,}020^3(1-0{,}3^4)} = 10.9109\ MPa$$

Principal stresses :

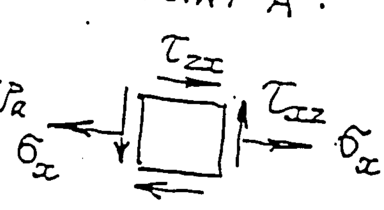
$$\sigma_1 ; \sigma_3 = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xz}^2} = \frac{7.6376}{2} \pm \sqrt{\left(\frac{7.6376}{2}\right)^2 + 10.9109^2}$$

$\sigma_1 = 3.8188 + 11.5599 = 15.3787\ MPa$

$\sigma_3 = 3.8188 - 11.5599 = -7.7411\ MPa$

$\sigma_2 = 0$ $\quad \tau_{max} = \frac{1}{2}(\sigma_1 - \sigma_3) = 11.5599\ MPa$

POINT A :



ANS

Stresses at B:

$$\tau'_{xy} = \tau_{xz} = 10.9109\ MPa$$

$$\tau''_{xy} = \frac{QV}{Ib}$$

$$V = 850\ N$$

$$I = \frac{\pi}{64} D^4(1-\psi^4) = \frac{\pi}{64} 0.02^4 (1-0.3^4) = 7.7904\ (10^{-9})\ m^4$$

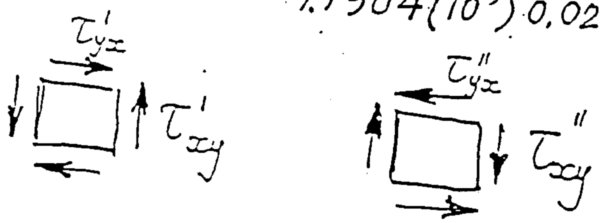
$$Q = y_c A = \frac{4}{3\pi} R \frac{\pi}{2} R^2 - \frac{4}{3\pi} r \frac{\pi}{2} r^2 = \frac{4}{6}(R^3 - r^3)$$

$$Q = \frac{4}{6}(0.01^3 - 0.003^3) = 648.667(10^{-9})\ m^3$$

$$Q = \frac{4}{6}(R^3 - r^3) = \frac{4}{6}(R^3 - r^3)\ ; \quad b = 2R = 0.020\ m$$

$$\tau''_{xy} = \frac{4(4)(R^3 - r^3)V}{3\pi(R^4 - r^4)2R} = \frac{16V(R^3 - r^3)}{6\pi(R^4 + r^4)R} = \frac{8V(R^3 - r^3)}{3\pi(R^4 + r^4)R}$$

$$\tau''_{xy} = \frac{648.667(10^{-9})\,85}{7.7904(10^{-9})\,0.02} = 0.3539\ MPa$$



$$\tau_{xy} = \tau'_{xy} - \tau''_{xy} = 10.9109 - 0.3539 = 10.557\ MPa$$

Principal stresses

$$\sigma_1 = -\sigma_3 = \tau_{xy} = \tau_{max} = 10.557\ MPa$$

$$\sigma_2 = 0$$

Problem 5

$$\sigma_1^2 - \sigma_1\sigma_3 + \sigma_3^2 = S_y^2 \quad (1)$$

$$\sigma_{1,3} = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$ When this substituted

into (1), left side of (1) becomes

$$\Big[\overset{A}{\frac{\sigma_x+\sigma_y}{2}} + \overset{B}{\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}}\Big]^2 - \Big[\frac{\sigma_x+\sigma_y}{2} + \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}\Big]\Big[\frac{\sigma_x+\sigma_y}{2} - \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}\Big]$$
$$+ \Big[\frac{\sigma_x+\sigma_y}{2} - \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}\Big]$$

$$(A+B)^2 - (A+B)(A-B) + (A-B)^2 =$$
$$A^2 + 2AB + B^2 - A^2 + B^2 + A^2 - 2AB + B^2 = A^2 + 3B^2$$
$$\left(\frac{\sigma_x+\sigma_y}{2}\right)^2 + 3\Big[\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2\Big] = \frac{1}{4}\left(\sigma_x^2 + 2\sigma_x\sigma_y + \sigma_y^2\right) +$$
$$+ \frac{3}{4}\left(\sigma_x^2 - 2\sigma_x\sigma_y + \sigma_y^2\right) + 3\tau_{xy}^2 = \sigma_x^2 - \sigma_x\sigma_y + \sigma_y + 3\tau_{xy}^2$$

Finally $\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2 = S_y^2$

ANS

Problem 6

$$\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2 = S_y^2$$ von Mises criterion

$$\sigma_x^2 + 3\tau_{xy}^2 = S_y^2 \;; \quad \sigma_y = 0 \qquad (1)$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{S_y}{2}$$ max shear failure criterion

Squaring and taking $\sigma_y = 0$

$$\frac{\sigma_x^2}{4} + \tau_{xy}^2 = \frac{S_y^2}{4} \Rightarrow \sigma_x^2 + 4\tau_{xy}^2 = S_y^2$$

What is the maximum shear failure criterion for bending and torsion of a component. It is more conservative then the von Mises criterion (1).

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ASSIGNMENT 2

Due date: Monday, October 14, 2002

1: A 6 in diameter steel shaft is to have a press fit with a 12 in outside diameter cast iron hub. Both the hub and the shaft are 10 in long. The maximum circumferential stress is to be 5000 psi. The moduli of elasticity are 30×10^6 psi for steel and 15×10^6 for cast iron. The Poison's ratio for both steel and cast iron is 0.3 and the coefficient of friction for the two materials is 0.12. Determine:

a) The maximum permissible radial interference
b) The axial force required to press the hub on the shaft
c) What torque this press fit can transmit

2: A rectangular cross section of a curved member has the width of b=1 in and height of h=3 in and is subjected to a pure positive moment (tension of the inner fiber) of 20000 lbf-in. No other type of loading is acting on the member. Find the maximum stress for the following geometries:

a) A straight member
b) A member whose centroidal axis has a radius of 15 in.
c) A member whose centroidal axis has a radius of 3 in.

3: Problem 5-20 (Page 306)	From the textbook
4: Problem 5-32 (Page 310)	From the textbook
5: Problem 5-60 (Page 314)	From the textbook
6: Problem 5-62 (page 315)	From the textbook

The problems are equally valued at 5 marks for a total of 30 marks.

MECH 441

Assignment #2, Fall 2002

PROBLEM 1:

$r_i = 0$ $R = 3$ in (interference Radius)

$r_o = 6$ in

$E_i = 30 \times 10^6$ psi $\nu_i = 0.3$

$E_o = 15 \times 10^6$ psi $\nu_o = 0.3$

$l = 10$ in $f = 0.12$

$\sigma_{t\,max} = 5000$ psi

$\sigma_{i\,max} = 5000$ psi

$$\sigma_{t_{i\,max}} = -\frac{1+\psi_i^2}{1-\psi_i^2} p \rightarrow p = -\frac{1-\psi_i^2}{1+\psi_i^2}\sigma$$

$$p = -\frac{1-0}{1+0} 5000 = 5000\,p$$

a) interference pressure is

$$p = \frac{\sigma_{t\,max}(r_o^2 - R^2)}{r_o^2 + R^2} = \frac{5000 \times (6^2 - 3^2)}{6^2 + 3^2} = 3000 \text{ psi}$$

choose smaller p

$p = 3000$ psi

the maximum permissible radial interference is:

$$\delta = \frac{pR}{E_o}\left(\frac{r_o^2 + R^2}{r_o^2 - R^2} + \nu_o\right) + \frac{pR}{E_i}\left(\frac{R^2 + r_i^2}{R^2 - r_i^2} - \nu_i\right)$$

After substituting for p; R, r_o; r_i, E_i, E_o; ν_i; ν_o we have:

$\delta = 1.390 \times 10^{-3}$ in

b) the force required for press fit is:

$$F_{max} = pf(2\pi R l) = 3000 \times 0.12(2\pi \times 3 \times 10) \cong 67860 \text{ lbf}$$

c) the torque is

$$T = RF_{max} = 3 \times 67860 \cong 203600 \text{ lbf-in}$$

PROBLEM 2:

a) for a straight beam $I = \frac{bh^3}{12}$, $c = h/2$ $Z = \frac{bh^2}{6}$

$\sigma = \frac{M}{Z} = \frac{20000 \times 6}{1 \times 3^2} = 13333$ psi $\Rightarrow \begin{cases} \sigma_i = 13333 \text{ psi inner fiber} \\ \sigma_o = -13333 \text{ psi outer fiber} \end{cases}$

b) The outer and inner radii relative to the centroidal radius of 15 in are:

$r_o = r_c + h/2 = 15 + 3/2 = 16.5$ in $\quad r_i = r_c - h/2 = 15 - 3/2 = 13.5$ in

Then the neutral radius is:

$r_n = \frac{r_o - r_i}{\ln \frac{r_o}{r_i}} = \frac{3}{\ln \frac{16.5}{13.5}} = 14.95$ in $\Rightarrow e = r_c - r_n = 15 - 14.95 = 0.05$ in

Now the distances from the neutral axis to the inner and outer fibers are:

$c_o = h/2 + e = 1.5 + 0.05 = 1.55$ in $\quad c_i = h/2 - e = 1.5 - 0.05 = 1.45$ in

The corresponding normal stresses are: $\sigma_i = \frac{Mc_i}{Ae r_i} = \frac{20000 \times 1.45}{3 \times 1 \times 0.05 \times 13.5} \Rightarrow$

$\sigma_i = +14321$ psi (tension) $\quad \sigma_o = \frac{-Mc_o}{Aer_o} = \frac{-20000 \times 1.55}{3 \times 1 \times 0.05 \times 16.5} = -12525$ psi (compression)

Because the radius of curvature is so large, there is little difference from the stresses found for a straight beam in part (a)

c) The outer and inner radii relative to the centroidal radius of 3 in are: $r_o = r_c + h/2 = 3 + 3/2 = 4.5$ in $\quad r_i = 3 - 3/2 = 1.5$ in

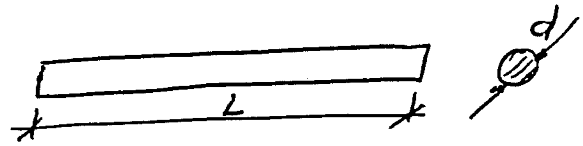
$r_n = \frac{r_o - r_i}{\ln r_o/r_i} = \frac{4.5 - 1.5}{\ln 4.5/1.5} = 2.73$ in $\Rightarrow e = r_c - r_n = 3 - 2.73 = 0.27$ in

$c_o = h/2 + e = 1.5 + 0.27 = 1.77$ in, $\quad c_i = 1.5 - 0.27 = 1.23$ in

$\Rightarrow \sigma_i = +\frac{Mc_i}{Aer_i} = + \frac{20000 \times 1.23}{1 \times 3 \times 0.27 \times 1.5} = 20247$ psi, $\quad \sigma_o = -\frac{Mc_o}{Aer_o} = -\frac{20000 \times 1.77}{1 \times 3 \times 0.27 \times 4.5}$

$\Rightarrow \sigma_o = -9712$ psi

3- PROBLEM 5-20 Text book



$N = 2$ Safty factor

$T = 100$ N-m , $L = 1$ m $\quad \theta_{max} = 2^{\circ} = \frac{2\pi}{180}$ rad $= 0.03491$ rad

Assume Ductile Material; $G = 79$ Gpa

$$\theta = \frac{TL}{JG} \Rightarrow J = \frac{TL}{\theta G} = \frac{100 \times 1}{0.03491 \times 79 \times 10^{9}} \Rightarrow$$

$J = 36.26 \times 10^{-9}\ m^4 = 3.626 \times 10^{4}\ mm^4$ Polar Moment of Inertia

$$J = \frac{\pi d^4}{32} \Rightarrow d = \left(\frac{32 J}{\pi}\right)^{1/4}$$

$d = \left(\frac{32 \times 3.626 \times 10^{4}}{\pi}\right)^{1/4} = 24.653^{mm}$ diameter of shaft

Determine the shear stress at the outside diameter of shaft;

$$\tau_{max} = \frac{T \cdot d/2}{J} = \frac{100 \times \frac{24.653}{2} \times 10^{-3}}{36.26 \times 10^{-9}} \cong 34\ MPa$$

Now use the distortion-energy theory:

$N = \frac{S_y}{\sigma'}$; $\sigma' = \sqrt{3}\,\tau_{max}$

Von-Mises stress

$\Rightarrow S_y = \sqrt{3}\,\tau_{max} \times N$ minimum required yield strength

$S_y \cong 118$

Any steel listed in table C-9 in Appendix C will be adequate.

(The least expensive is AISI 1010 hot-rolled)

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4 - PROBLEM 5-32 text book

Stresses : $\sigma_x = 10$ ksi $\quad \sigma_y = 5$ ksi $\quad \tau_{xy} = 4.5$ ksi

Strengths : $S_y = 18$ ksi $\quad S_{ut} = 20$ ksi $\quad S_{uc} = 80$ ksi

Because S_{uc} is greater than S_{ut}, this is an uneven material, which is characteristic of a brittle material. Therefore, we use modified-Mohr Theory

First obtain maximum shear stress and principal stresses ;

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 5.148 \text{ ksi}$$

$$\left.\begin{aligned} \sigma_1 &= \frac{\sigma_x + \sigma_y}{2} + \tau_{max} = 12.648 \text{ ksi} \\ \sigma_2 &= \frac{\sigma_x + \sigma_y}{2} - \tau_{max} = 2.352 \text{ ''} \end{aligned}\right\} \text{principal stresses}$$

$\sigma_3 = 0$

Find the dowlings factors using equations 5.12C :

$C_1 = 8.898$ ksi , $\quad C_2 = 1.764$ ksi , $C_3 = 9.486$ ksi

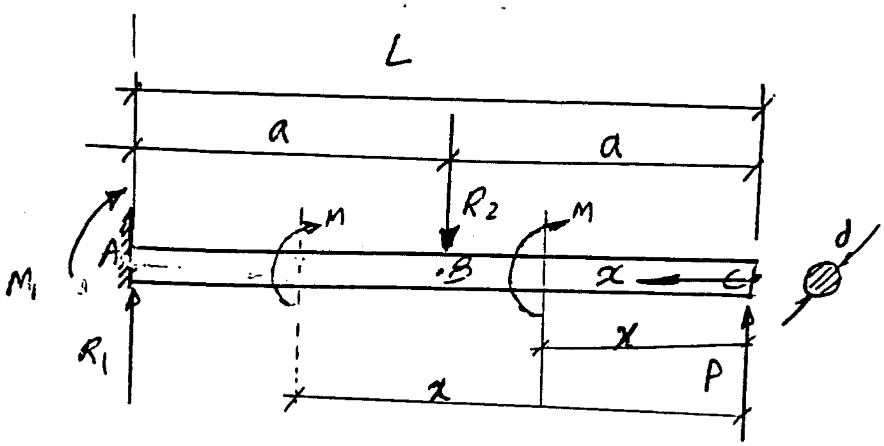
Now the modified-Mohr effective stress is :

$$\hat{\sigma} = \max(C_1, C_2, C_3, \sigma_1, \sigma_2, \sigma_3).$$

$$\Rightarrow \hat{\sigma} = 12.648 \text{ ksi}$$

Finally the safty factor $N = \dfrac{S_{ut}}{\hat{\sigma}} = 1.6$

5- PROBLEM 5-60 text book



$d = 1''$
$P = 500$ lbf
$a = 20''$
$L = 40''$
$E = 30 \times 10^6$ psi
$S_y = 54$ ksi
(Tensile yield strength. See Table C-9 in Appendix C)

This is statically indeterminate beam because there are three unknown reactions R_1, R_2 and M_1 and with two the equilibrium eqvations we cannot obtain them. The singularity functions may be used to obtain deflection, slope, moment and shear across the beam and after applying boundary conditions the unknown reactions can be obtained. Here we use energy method, just to show its efficiency in solving these types of ploblems.

According to Castigliano's Theorem for linearly elastic systems, we have:

$$\Delta = \frac{\partial U}{\partial P} \quad \text{in General}$$

Δ: deflection, U: strain energy

but $$U = \int_0^L \frac{M^2}{2EI}\,dx$$ strain energy due to bending

$$\Rightarrow \Delta_B = \frac{\partial U}{\partial R_2} = \int_0^L \frac{M}{EI}\frac{\partial M}{\partial R_2}\,dx \quad (*)$$

from E to B :

$M = Px$ and $\frac{\partial M}{\partial R_2} = 0$

from B to A

$M = Px - R_2(x-a)$ $\frac{\partial M}{\partial R_2} = -x+a$

$$\Delta_B = \frac{\partial U}{\partial R_2} = \frac{1}{EI}\int_0^{L/2} \cancel{(Px) \times 0 \, dx} + \frac{1}{EI}\int_{L/2}^{L} [Px - R_2(x-a)](-x+a)\,dx$$

$$\Rightarrow \Delta_B = \frac{1}{EI}\int_{L/2}^{L} [R_2(a^2 - 2ax + x^2) + Pax - Px^2]\,dx$$

$$\Rightarrow \Delta_B = \left[R_2\left(a^2 L/2 - 3aL^2/4 + 7L^3/24\right) + \frac{3PaL^2}{8} - \frac{7PL^3}{24}\right]/EI$$

but $\Delta_B = 0$

$$\Rightarrow R_2 = \frac{\frac{7PL^3}{24} - \frac{3PaL^2}{8}}{a^2 L/2 - 3aL^2/4 + 7L^3/24} = \frac{\frac{7\times500\times40^3}{24} - \frac{3\times500\times20\times40^2}{8}}{\frac{20^2\times40}{2} - \frac{3\times20\times40^2}{4} + \frac{7\times40^3}{24}} = 1250 \text{ lbf}$$

we have :

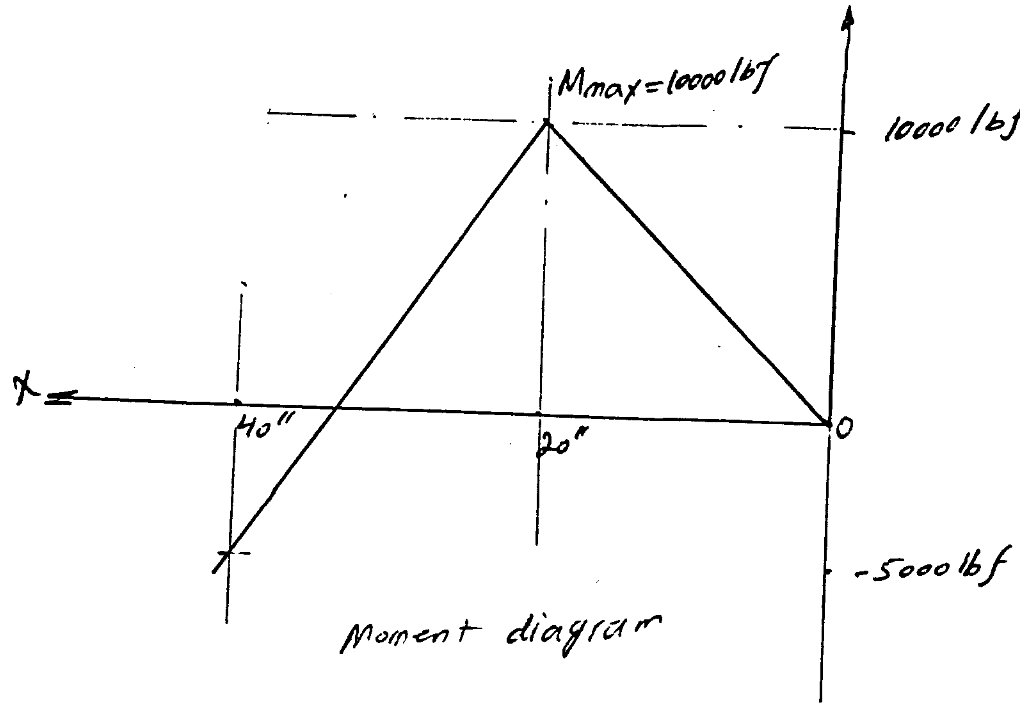
$R_1 - R_2 + P = 0 \Rightarrow R_1 = R_2 - P = 1250 - 500 = 750$ lbf

$M_1 = PL - R_2(L-a) = 500\times40 - 1250(40-20) = -5000$

negative means direction should be reversed

The maximum moment occurs at point B

$M_{max} = M_B = P\times a = 500\times20 = 10000$ lbf



Moment diagram

Now maximum bending stress can be detemined:

$$\sigma_{max} = \frac{M_{max} \cdot C}{I} \Rightarrow \quad \sigma_{max} = 101860 \text{ psi}$$

where $C = 0.5d = 0.5''$, $I = \frac{\pi d^4}{64} = 0.049087 \text{ in}^4$

There are no other stress components present (The transverse shear is zero at the extreme fiber) so this is a principal stress and the other two principal stresses are zero. Thus, this is a case of uniaxial stress.

$$N = \frac{S_y}{\sigma_{max}} = \frac{54000}{101860} = 0.53 < 1 \quad \text{failure will occur!}$$

N ↓ Safty factor

---

if you select a diameter of $d = 1.5''$ for the rod:

$C = 0.5d = 0.75''$, $I = \frac{\pi d^4}{64} = 0.2485 \text{ in}^4$

$$\Rightarrow \sigma_{max} = \frac{M_{max} \cdot C}{I} = \frac{10000 \times 0.75}{0.2485} = 30180 \text{ psi}$$

$$\rightarrow N = \frac{S_y}{\sigma_{max}} = \frac{54000}{30180} = 1.8 > 1 \quad \text{Now it is not going fail.}$$

6- PROBLEM 5-62 textbook

$F = 100$ lbf , SAE 1020 Cold-rolled steel: $S_y = 57$ ksi

$l = 2$ in , $d = 0.5$ in $\qquad L = 0.5l = 0.5 \times 2 = 1$ in

Total length, Pin diameter, Beam length

* Since there is a slip fit between the pin and part B, part B offers no resistance to bending of the pin and, because the pin is press-fit into part A, it can be modeled as a cantilever beam of length $l/2$

** parts B distributes the concentrated force F so that, at the pin, it is uniformly distributed over the exposed length of the pin.

$$q = \frac{F}{L} = \frac{100}{0.5 \times 2} = 100 \text{ lb/in}$$ → intensity of the uniformly distributed load acting over the length of the pin.

A Cantilever beam with uniform loading is shown in figure D-1(b) in Appendix D. In this case, the dimension a in the figure is zero. As shown in the figure, when $a=0$, the maximum bending moment occurs at the support and is:

$$M_{max} = \frac{qL^2}{2} = \frac{100 \times (0.5 \times 2)^2}{2} = 50 \text{ lbf-in}$$

Now the maximum bending stress in the beam is:

$$\sigma_{max} = \frac{M_{max} \cdot C}{I}$$, where $C = \frac{d}{2} = \frac{0.5}{2} = 0.25$ in

$I = \pi d^4/64 = 3.068 \times 10^{-3}$ in^4

Thus :

$$\sigma_{max} = \frac{50 \times 0.25}{3.068 \times 10^{-3}} \cong 4074 \text{ psi}$$

There are no other stress components present (the transverse shear is zero at the extreme fibers) so this is a principal stress and the other two principal stresses are zero. thus, this is a case of uniaxial stress :

$$N = \frac{S_y}{\sigma_{max}} = \frac{57000}{4074} \cong 14$$

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MECH 441 Assignment 3 Fall 2001

Problem 1 (P. 4-53)

Given : $L = 3\ m$ $F = 900\ N$
$t = 5\ mm$ $S_{yc} = 150\ MPa$
$N = 3$ $E = 71.7\ GPa$

$$S_{rD} = \pi \sqrt{\frac{2E}{S_{yc}}} = \pi \sqrt{\frac{2(71.7)10^9}{150\ 10^6}} = 97.136$$

(a) Pinned-pinned ends

$L_{eff} = L = 3\ m$; Assume the Euler's region

$$P_{cr} = \frac{\pi^2 E A k^2}{L^2}; \quad k^2 = \frac{I}{A}$$

$$P_{cr} = \frac{\pi^2 E I}{L};$$

$$P_{cr} = NF = 3(900) = 2700\ N$$

$$I = \frac{L^2 P_{cr}}{\pi^2 E} = \frac{3^2(2700)10^6}{\pi^2(71.7)10^9} = 34.33897851 0^{-3} m^2 = \frac{\pi}{64}\left[D^4 - (D-2t)^4\right]$$

$D = 30.63\ mm$

$$I = \frac{\pi}{64}(30.63^4 - 20.63^4) = 34316.043\ mm^4; \quad D = 30.64\ mm$$

$$A = \frac{\pi}{4}(D^2 - d^2) = \frac{\pi}{4}(30.64^2 - 20.64^2) = 402.75\ mm^2$$

$$k_r = \sqrt{\frac{I}{A}} = \sqrt{\frac{34355.24}{402.75}} = 9.236\ mm$$

$$S_r = \frac{L_{eff}}{k_r} = \frac{3000}{9.236} = 324.82 > 97.136$$ - Euler's region

$D = 30.64\ mm$ ANS

(b) Fixed-free ends

$L_{eff} = 2L = 6000\ mm$ - theoretical but not recommended value

$$P_{cr} = \frac{\pi^2 E I}{L_{eff}} = 3(900) = 2700\ N$$

$$I = \frac{L_{eff}^2 P_{cr}}{\pi^2 E} = \frac{6^2(2700)10^6}{\pi^2(71.7)10^9} = 137355.88 \ \ mm^4$$

$$I = \frac{\pi}{64}\left[D^4-(D-0.01)^4\right] = 137355.88 \ \ mm^4$$

$$D = 46.00 \ \ mm$$

$$I = \frac{\pi}{64}\left(46.00^4-36.00^4\right) = 137338.64 \ \ mm^2 \Rightarrow D = 46.0 \ mm$$

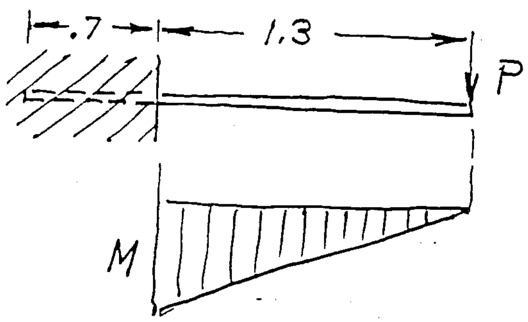
$$A = \frac{\pi}{4}\left(D^2-d^2\right) = \frac{\pi}{4}\left(46.0^2-36.0^2\right) = 644.0265 \ \ mm^2$$

$$k_r = \sqrt{\frac{I}{A}} = \sqrt{\frac{137338.64}{644.0265}} = 14.603 \ \ mm$$

$$S_r = \frac{L_{eff}}{k_r} = \frac{6000}{14.603} = 410.8 > S_{rD} = 97.136 \ \ \text{Euler's region}$$

Therefore $D = 46.00 \ mm$ <u>ANS</u>

Problem 2 (P. 5-12)



Given: $W = 305 \ mm$; $t = 32 \ mm$

$W = 100 \ kg$; $S_{ut} = 130 \ MPa$

$P = Mg = 100(9.81) = 981 \ N$

$M = P(1.3) = 981(1.3) = 1275.3 \ Nm$

$$\sigma_x = \frac{6M}{Wt^2} = \frac{6(1275.3)}{0.305(0.032)^2} = 24.5 \ MPa = \sigma_1$$

$$\sigma_2 = \sigma_3 = 0$$

$$N_S = \frac{S_{ut}}{\sigma_1} = \frac{130}{24.5} = 5.306$$ <u>ANS</u>

Problem 3 (P. 5-22)

Given: $m = 100\ kg$

$m_b = 2\ kg$

$k = 6000\ N/m$

$h = 0.5\ m$

$$\eta = \frac{1}{1+\frac{m_b}{3m}} = \frac{1}{1+\frac{2}{3(100)}} = 0.9933$$

$$\delta_{st} = \frac{W}{K} = \frac{mg}{K} = \frac{100(9.81)}{6000} = 0.1635\ m$$

$$F_i = \left(1+\sqrt{1+\frac{2\eta h}{\delta_{st}}}\right)W = \left(1+\sqrt{1+\frac{2(0.9933)0.5}{0.1635}}\right)(981)$$

$$F_i = 3.6599(981) = 3590.39\ N$$

(a) $$\tau_a = \frac{4F_i}{4(2)\pi d^2} = \frac{3590.39}{2\pi(0.01)^2} = 5.7143\ MPa$$

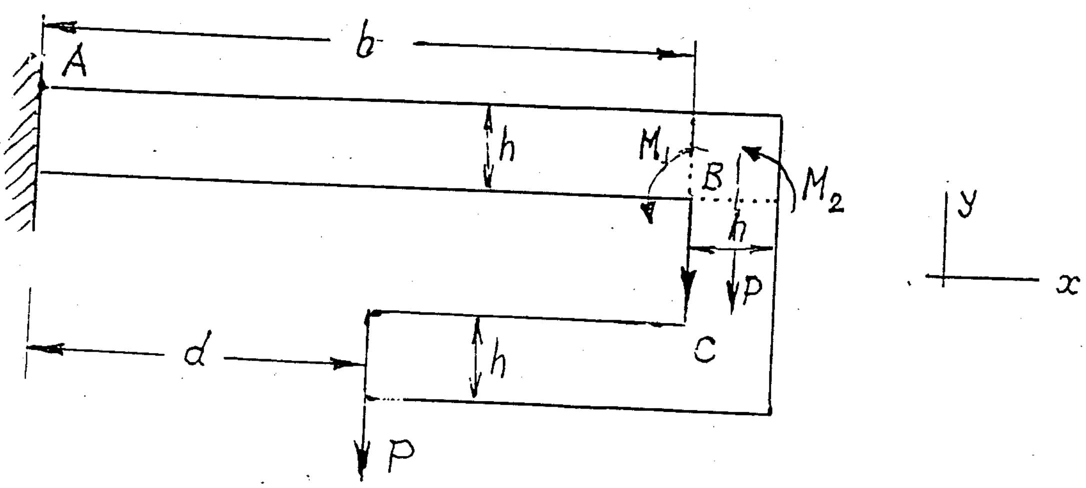
$$N_a = \frac{S_y}{\sqrt{3}\,\tau_a} = \frac{400}{\sqrt{3}(5.7143)} = 40.41 \qquad \underline{\text{ANS}}$$

(b) $$\tau_b = \frac{4F_i}{2\pi d^2} = \frac{400}{2\pi(0.01)^2} = 22.8572\ MPa$$

$$N_b = \frac{S_y}{\sqrt{3}\,\tau_b} = \frac{400}{\sqrt{3}(22.8572)} = 10.10 \qquad \underline{\text{ANS}}$$

Problem 4 (P.5-59)

Given: $d = 8$ in ; ; $b = 17$ in ; $h = 3$ in
$t = 1.0$ in ; $S_{ut} = 62$



Point A: $M_A = Pd = 5000(8) = 40\,000$ lb/in

$$\sigma_{Ax} = -\frac{6M_A}{th^2} = -\frac{6(40000)}{1.0(3)^2} = -26.667 \text{ MPa}$$

$$N_s = \frac{S_{uc}}{\sigma_{Ax}} = \frac{187}{26.667} = 7.0$$

ANS

Point B: $M_{B1} = P(b-d) = 5000(17-8) = 45000$ lbin

$$\sigma_{Bx} = \frac{6M_{B1}}{th^2} = \frac{6(45000)}{1.0\,(3)^2} = 30000 \text{ ksi}$$

$$M_{B2} = P\left(b+\frac{h}{2}-d\right) = 5000\left(17+\frac{3}{2}-8\right) = 52500 \text{ lbin}$$

$$\sigma_{By} = \frac{6M_{B2}}{th^2} + \frac{P}{th} = \frac{6(52500)}{1.0(3)^2} + \frac{5000}{1.0(3)} = 35000 + 1666.7 \text{ ksi}$$

$\sigma_{By} = 36.667$ ksi $\qquad \tau_{Bxy} = 0$

$\sigma_1 = \sigma_{By} = 36.667$ ksi ; $\sigma_2 = \sigma_{Bx} = 30.000$ ksi ; $\sigma_3 = 0$

$$N_s = \frac{S_{ut}}{\sigma_1} = \frac{62.000}{36.667} = 1.69$$

ANS

Problem 5 (P. 7-19)

Given: $D_1 = 40$ mm ; $R_1 = 20$ mm ; $E_1 = E_2 = 206.8$ GPa
$D_2 = 50$ mm ; $R_2 = 25$ mm ; $\nu_1 = \nu_2 = 0.28$
$L = 250$ mm ; $F = 10000$ N

$$p_{max} = 0.564\sqrt{\frac{F\left(\frac{1}{R_1}+\frac{1}{R_2}\right)}{Lm}}$$

$$m_1 + m_2 = m = \frac{2(1-\nu^2)}{E} = \frac{2(1-0.28^2)}{206.8(10^9)} = 8.91296(10^{-12})\ Pa^{-1}$$

$$p_{max} = 0.564\sqrt{\frac{10000\left(\frac{1}{0.02}+\frac{1}{0.025}\right)}{0.25(8.91296)}\cdot 10^6} = 358.44(10^6)\ N/m^2$$

$p_{max} = 358.44$ MPa ANS

$\sigma_{z\,max} = -p_{max} = -358.44$ MPa $= \sigma_{x\,max}$ ANS

$\sigma_{y\,max} = -2\nu\, p_{max} = -2(0.28)358.44 = -200.726$ MPa ANS

$\tau_{max} = \frac{1}{2}|\sigma_3 - \sigma_1| = \frac{1}{2}|-358.44-(-200.726| = 78.857$ MPa ANS on surface

$\sigma_{z\,max} = \sigma_{x\,max} = \sigma_2 = \sigma_3$; $\sigma_{y\,max} = \sigma_1$

$z_{max} = 0.786a = 0.786(0.07101) = 0.0558$ mm ; $\tau_{max} = 0.3 p_{max} = 107.53$ MPa

$$a = \sqrt{\frac{2mF}{\pi B L}} = \sqrt{\frac{2(8.91296)10^{-12}}{\pi(45)0.25}} = 0.7102(10^{-4})\ m = 0.07102\ mm$$

$$B = \frac{1}{2}\left(\frac{1}{R_1}+\frac{1}{R_2}\right) = \frac{1}{2}\left(\frac{1}{0.02}+\frac{1}{0.025}\right) = 45\ m^{-1}$$

Problem 6 (P. 9.11)

Given $D = 6.00$ in ; $E = 30(10^6)$ psi

$d = 1.75$ in ; $\nu = 0.28$

$L = 1.0$ in ; $S_{ut} = 108$ ksi ; $S_y = 62$ ksi

$T_{max} = 2000$ lbin

$$\psi_e = \frac{d}{D} = \frac{1.75}{6.0} = 0.2917$$

$$\psi_i = \frac{0}{d} = 0$$

Minimum pressure and interference from the torque :

$$T_{max} = \frac{1}{2} d^2 \pi L \mu p_{min} ;$$

$$p_{min} = \frac{2T_{max}}{d^2 \pi L \mu} = \frac{2(2000)}{1.75^2(\pi)1.0(0.15)} = 2771.678 \text{ psi}$$

$$\delta_{min} = \frac{p_{min} d}{E} \frac{1-\psi_e^2\psi_i^2}{(1-\psi_i^2)(1-\psi_e^2)} = \frac{2771.678(1.75)}{30(10^6)} \frac{1-0}{(1-0)(1-0.2917^2)}$$

$$\delta_{min} = 176.71795(10^{-6}) = 0.000177 \text{ in on radius}$$

$\Delta_{min} = 0.000354$ in on diameter. Applying a design saf
factor $N_d = 2$ we have

$$\Delta_{min} = 2(0.000354) = 0.00071 = 0.0007 \text{ in - rounded off}$$

Stresses in shaft

$$\sigma_r = -p_{max}$$

$$\sigma_t = -\frac{1+\psi_i^2}{1-\psi_i^2} p_{max} = \frac{1+0}{1-0} p_{max} = -p_{max} ; \sigma' = \sqrt{(-1)^2+(-1)^2-(-1)(-1)}\, p_{max}$$

$$\sigma' = p_{max}$$

Stresses in hub

$$\sigma_r = -p_{max}$$

$$\sigma_t = \frac{1+\psi_e^2}{1-\psi_e^2} p_{max} = \frac{1+0.2917^2}{1-0.2917^2} p_{max} = 2.093\, p_{max}$$

$$\sigma' = \sqrt{2.093^2+(-1)^2-(2.093)(-1)}\, p_{max} = 2.7338\, p_{max}$$

$$N_d = \frac{S_y}{\sigma'} = \frac{S_y}{2.7338\, p_{max}}; \quad p_{max} = \frac{S_y}{2.7338\, N_d}$$

$$p_{max} = \frac{62000}{2.7338(2)} = 11339.527 \text{ psi}$$; $N_d = 2$ design safety factor

$$\Delta_{max} = \frac{p_{max}}{p_{min}} \Delta_{min} = \frac{11339.527}{2771.678}\, 0.0007 = 0.00029 \text{ in}$$

p_{max} from the stresses in shaft $\sigma' = p_{max}$

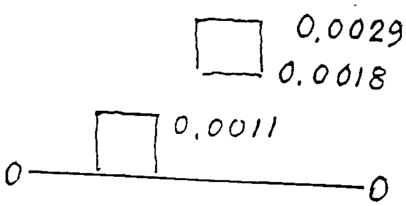
$$p_{max} = \frac{S_y}{N_d} = \frac{62000}{2} = 31000 \text{ psi}$$ which is higher than from the stresses in the hub, hence stress in the hub is more critical.

Hence, the interference limits are

$\Delta_{max} = 0.0029$ in
$\Delta_{min} = 0.0007$ in
} interference limits on diameter

Take tolerances equal $T = t = \frac{1}{2}(\Delta_{max} - \Delta_{min})$

$$= \frac{1}{2}(0.0029 - 0.0007) = 0.0011 \text{ in}$$



$d_{max} = 1.7529''$
$d_{min} = 1.7518''$
} shaft limits

$D_{max} = 1.7511''$
$D_{min} = 1.7500''$
} Hub bore limits

The design safety factor is 2 for both stresses in the hub and with respect to torque rating of 2000 lbin.

.24

Problem 6 (P14-25) row(l)

Given: $L = 150\ mm$; $a = 250\ mm$; $t = 8\ mm$; $F = 750\ N$

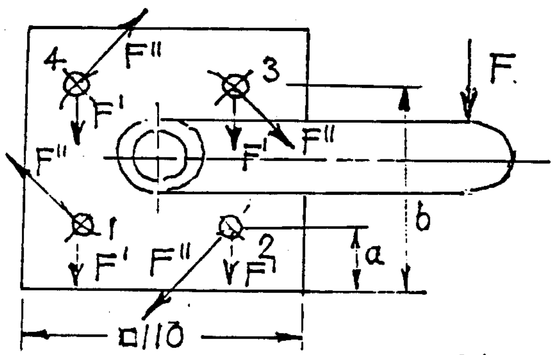
$d_o = 45\ mm$; $d_i = 30\ mm$; aluminum ; $D_{bc} = 100\ mm$

Try M8 cap screw ; $E = 71.8\ GPa$; $N_b = 4$

M8 class 9.8 ; $S_p = 650\ MPa$; $S_y = 720\ MPa$; $S_{ut} = 900\ MPa$; $E = 207\ GPa$

Solution

$F_{max} = 750\ N$; $F_{min} = 0$; $N = 1.5$; $N = 5(10^8)$ cycles



Torque: $T = Fa = 750(0.25) = 187.5\ Nm$

Moment: $M = F(L+t) = 750(0.15+0.008)$

$M = 118.5\ Nm$

Direct shear force

$$F_s' = \frac{750}{4} = 187.5\ N$$

Shear force due to the torque

$$F_s'' = \frac{2T}{N_b D_{bc}} = \frac{2(187.5)}{4(0.1)} = 937.5\ N$$

Tension force in bolts:

$$2F_{1,2}a + 2F_{3,4}b = M$$

Take a square $s = 110\ mm$; then

$a = 55 - 50\cos 45° = 19.64\ mm$

$b = a + 2(50)\cos 45° = 90.35\ mm$

$$2F_{1,2}(19.64) + 2F_{3,4}(90.35) = 118.5(1000)$$

$$F_{1,2} + 4.60\,F_{34} = 4047.13$$

But also for a perfectly rigid plate, the bolt deflections are

$$\delta_{1,2} = \frac{19.64}{90.35}\delta_{3,4}$$; According to the Hook's law the forces are proportional to the deflections

$$F_{1,2} = \frac{19.64}{90.35}F_{3,2} = 0.2174\,F_{3,4}$$

$0.2174 F_{3,4} + 4.60 F_{3,4} = 4047.13 \Rightarrow F_{3,4} = 840.11$ N ; $F_{1,2} = 182.64$ N

The shear forces can be found using the cosine theorem :

$$F_{S1} = F_{S4} = \sqrt{(F_{S1}')^2 + (F_{S1}'')^2 - (F_{S1}')(F_{S1}'')\cos 45^\circ}$$
$$= \sqrt{187.5^2 + 937.5^2 - 187.5(937.5)\cos 45^\circ} = 888.69 \text{ N}$$
$$F_{S2} = F_{S3} = \sqrt{(F_{S2}')^2 + (F_{S2}'')^2 - (F_{S2}')(F_{S2}'')\cos 135^\circ}$$
$$= \sqrt{187.5^2 + 937.5^2 - 187.5(937.5)\cos 135^\circ} = 1018.0 \text{ N}$$

Screws 2&3 are loaded with a larger shear force of 1018.0 N and screws 3&4 are loaded with a larger tension force of 840.11 N. Therefore, screw 3 is the most loaded screw, the loads being $P = 840.11$ N and $F_S = 1018.0$ N.

Assume a screw length $l = t + 10p = 8 + 10(1.25) = 20.5$ mm
Assume 6 mm not threaded length $l_t = 20.5 - 6 = 14.5$ mm
The screw stiffness:

$$\frac{1}{k_b} = \frac{1}{k_1} + \frac{1}{k_2} = \frac{0.0145}{36.61(10^{-6})\,207(10^9)} + \frac{0.006}{\pi \frac{0.008^2}{4}(207)10^9}$$
$$\frac{1}{k_b} = 1.91332(10^{-9}) + 0.57665(10^{-9}) = 2.48997(10^{-9})$$
$$k_b = 4.0161(10^8) \text{ N/m}$$
$$d_2 = 1.5d = 1.5(8) = 12 \text{ mm}$$
$$d_3 = d_2 + 20.5\tan 42^\circ = 12 + 18.46 = 30.45 \text{ mm}$$
$$A_m = \frac{\pi}{4}(D_{eff}^2 - d^2) = \frac{\pi}{4}\left[\left(\frac{12+30.46}{2}\right)^2 - 8^2\right] = 303.724 \text{ mm}^2$$
$$k_m = \frac{A_m E_m}{l} = \frac{303.724(71.8)10^9}{20.5(10^{-3})} = 10.6377(10^8) \text{ N/m}$$
$$C = \frac{k_b}{k_b + k_m} = \frac{4.0161}{4.0161 + 10.6377} = 0.274$$
$$C - 1 = 1 - 0.274 = 0.726$$

$$P_b = CP = 0.274(840.11) = 230.19 \ N$$

$$P_m = (1-C)P = 0.726(840.11) = 609.92 \ N$$

$$F_i = 0.5\,F_p = 0.5\,(36.61)10^{-6}(650)10^{6} = 11898.25 \ N$$

$$F_b = F_i + P_b = 11898.25 + 230.19 = 12128.44 \ N$$

$$F_m = F_i - P_m = 11898.25 - 609.92 = 11288.33 \ N$$

$$P_o = \frac{F_i}{1-C} = \frac{11898.25}{0.726} = 16388.77 \ N$$

$$F_a = \frac{1}{2}(F_b - F_i) = \frac{1}{2}(12128.44 - 11898.25) = 115.095 \ N$$

$$F_{mean} = \frac{1}{2}(F_b + F_i) = \frac{1}{2}(12128.44 + 11898.25) = 12013.345 \ N$$

$$\sigma_a = K_f \frac{F_a}{A_t} = 3.0\,\frac{115.095}{36.61(10^{-6})} = 9.431 \ MPa$$

$$\sigma_{max} = \frac{F_b}{A_t} = \frac{12128.44}{36.61(10^{-6})} = 331.29 \ MPa$$

$$K_f\,|\sigma_{max}| = 3.0\,|331.29| = 993.87 > S_y = 720 \ MPa$$

$$K_{fm} = \frac{S_y - \sigma_a}{|\sigma_{max}|} = \frac{720 - 9.431}{331.29} = 2.145$$

$$\sigma_i = K_{fm}\frac{F_i}{A_t} = 2.145\,\frac{11898.25}{36.61(10^{-6})} = 697.125 \ MPa$$

$$\sigma_{mean} = K_f \frac{F_{mean}}{A_t} = 2.145\,\frac{12013.345}{36.61(10^{-6})} = 703.87 \ MPa$$

$$S_e = 0.5\,S_{ut} = 0.5(900) = 450 \ MPa$$

$$C_{load} = 0.7 \ ; \quad C_{surf} = 4.51\,S_{ut}^{-0.265} = 4.51(900)^{-0.265} = 0.744$$

$$C_{size} = 1.0 \ ; \quad C_{temp} = 1.0 \ ; \quad C_{reliab} = 0.814$$

$$S_e = C_{load}\,C_{size}\,C_{surf}\,C_{temp}\,C_{reliab}\,S_e'$$

$$= 0.7(1.0)(0.744)1.0(0.814)\,450 = 190.77 \ MPa$$

$$N_f = \frac{S_e(S_{ut} - \sigma_i)}{S_e(\sigma_{me} - \sigma_i) + S_{ut}\,\sigma_a} = \frac{190.77(900 - 695.125)}{190.77(703.87 - 697.125) + 900(9.431)} = 2.14 \quad \underline{ANS}$$

$$N_{sep} = \frac{P_o}{P} = \frac{16388.77}{840.11} = 19.50 \ ; \quad N_y = \frac{S_y - \sigma_i}{\sigma_{me} - \sigma_i + \sigma_a} = \frac{720 - 695.125}{703.87 - 697.125 + 9.431}$$

$$N_{fric} = \frac{\mu F_m}{F_{smax}} = \frac{0.15(11283.33)}{1018.0} = 1.66 > 1.5 \qquad N_y = 1.54 \quad \underline{ANS}$$