Concordia
UNIVERSITY

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| COURSE ~~Probability and Random~~ Processes in Engineering | NUMBER ENGR 371 | SECTION T,X | |
| EXAMINATION FINAL | DATE April 25, 1995 | TIME 3 hrs | # OF PAGES 5 |
| INSTRUCTOR Drs. M.Mehmet Ali, Hanqing Wu | | | |

MATERIALS ALLOWED: ☑ NO ☐ YES (PLEASE SPECIFY)

CALCULATORS ALLOWED: ☐ NO ☑ YES

SPECIAL INSTRUCTIONS:

1) Each front tire on a particular type of automobile is supposed to be filled to a pressure of 26psi. Suppose the actual air pressure in each tire is a random variable $X$ for the right tire and $Y$ for the left tire, with joint pdf

$$f(x,y)=\begin{cases} K(x^2+y^2) & 20<x<30,\ 20<y<30 \\ 0 & \text{otherwise} \end{cases}$$

i) What is the value of $K$?

ii) What is the probability that both tires are underfilled?

2) A library employee shelves 1000 books on a particular day. If the probability that any particular book is misshelved is 0.001 and books are shelved independently of one another, calculate the probability that

i) At least one book is misshelved on that day.

ii) Exactly one book is misshelved during a five-day workweek, assuming that what occurs on any one day is independent of what happens on any other day and 1000 books per day are shelved.

3) A particular type of gasoline tank for a compact car is designed to hold 15 gal. Suppose the actual capacity $X$ of a random chosen tank of this type is normally distributed with mean 15 gal and standard deviation 0.2 gal.

i) What is the probability that a randomly selected tank will hold between 14.7 and 15.1 gal?

ii) If the car on which a randomly chosen tank is mounted gets exactly 25 mile per gal., what is the probability that the car can travel 370 miles without refuelling?

4) Two random variables $X$ and $Y$ have a joint probability density function

$$f(x,y) = \begin{cases} \frac{3}{16}x^2 y & 0<x<2, \quad 0<y<2 \\ 0 & \text{elsewhere} \end{cases}$$

Find the probability distribution for $W = XY$.

5) The breaking strength of a rivet has a mean value of 10,000 psi and a standard deviation of 500 psi.

i) What is the probability that the sample mean breaking strength for a random sample of 40 rivets is between 9900 and 10,200?

ii) Assume that 160 independent random samples of size 40 are taken; find the probability that at least 140 of the sample means will fall between 9900 and 10,200.

6) Assume that the helium porosity (in percentage) of coal samples taken from any particular seam is normally distributed with a population standard deviation of 0.75.

i) Compute a 95% confidence interval for the mean porosity of a certain seam if the average porosity for 20 specimens from the seam is 4.85.

ii) How large a sample size is necessary if the length of the 95% confidence interval analogous to that of part i) is to be 0.40?

7) Consider the random process,

$$z(t) = t\, x \cos(wt + \theta)$$

where $X$ is a random variable with mean $\mu$ and variance $\sigma^2$. $\theta$ is a uniformly distributed random variable in the interval $(0, 2\pi)$ and independent of $X$.

i) Determine the autocorrelation function of the process $Z(t)$.

ii) Determine the variance of the process $Z(t)$.

iii) Is the process $Z(t)$ stationary, why?

$n = \left(\frac{Z_{\alpha/2}\,\sigma}{e}\right)^2$