ENGR370 MODELING AND ANALYSIS OF LINEAR PHYSICAL SYSTEMS – Test No.1 – Winter 2000

Time: One hour

1. Fig.1 shows the graph of a network.
   (a) List all the trees of the network.
   (b) Taking 24 as the tree and node(c) as the reference, write the matrices [A] and [B].
   (c) Verify $[A].[B]^T = 0$. (2 + 3 + 2 = 7 marks)

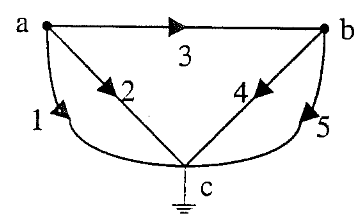


Fig.1

2. The following experimental data have been obtained in respect of two elements. Identify each element and obtain its magnitude and units. Give reasons for your answers. (3 + 3 = 6 marks)

Element A

Torque N.cm	Angular velocity (radians/second)
50	1
100	2
150	3

Element B

Force, N	Time, seconds	Velocity, m/s
8	0	0
8	1	3
8	2	6
8	3	9

3. Fig.3 shows an electrical network. Obtain the analogous mechanical (translational) circuit. Also, write the circuit giving the impedances and other elements in the Laplace transform domain.(Give reasons for your answers). (4 + 3 = 7 marks)

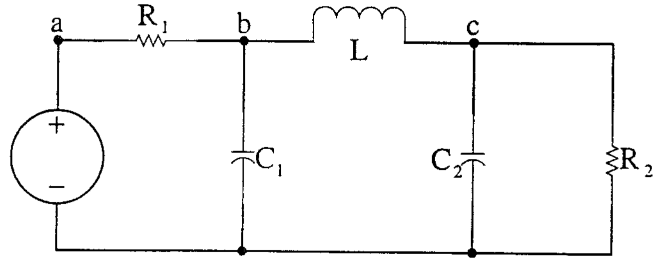
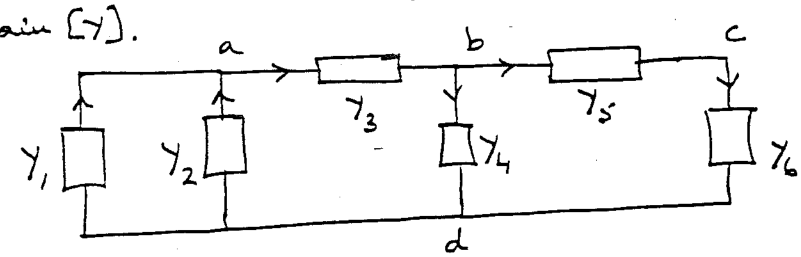


Fig.3.

(1) For the network shown below, obtain the node-admittance matrix [in the form $[A_3][Y_B][A_3]^T$. (Take d as the reference node) Hence obtain $[Y]$.



Solution:

$$A_3 = \begin{array}{c|cccccc} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline a & -1 & -1 & 1 & 0 & 0 & 0 \\ b & 0 & 0 & -1 & 1 & 1 & 0 \\ c & 0 & 0 & 0 & 0 & -1 & 1 \end{array}$$

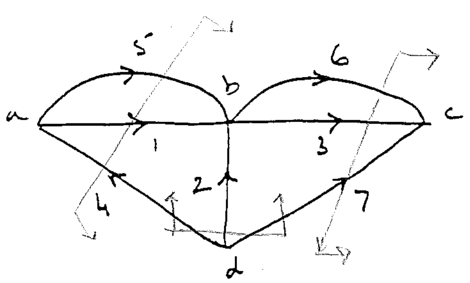
$$[Y_B] = \begin{array}{c|cccccc} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & Y_1 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & Y_2 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & Y_3 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & Y_4 & 0 & 0 \\ 5 & 0 & 0 & 0 & 0 & Y_5 & 0 \\ 6 & 0 & 0 & 0 & 0 & 0 & Y_6 \end{array}$$

$$[A_3][Y_B] = \begin{bmatrix} -Y_1 & -Y_2 & Y_3 & 0 & 0 & 0 \\ 0 & 0 & -Y_3 & Y_4 & Y_5 & 0 \\ 0 & 0 & 0 & 0 & -Y_5 & Y_6 \end{bmatrix}$$

$$[A_3][Y_B][A_3]^T = \begin{bmatrix} -y_1 & -y_2 & y_3 & 0 & 0 & 0 \\ 0 & 0 & -y_3 & y_4 & y_5 & 0 \\ 0 & 0 & 0 & 0 & -y_5 & y_6 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ -1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (y_1 + y_2 + y_3) & -y_3 & 0 \\ -y_3 & (y_3 + y_4 + y_5) & -y_5 \\ 0 & -y_5 & y_5 + y_6 \end{bmatrix}$$

(2) In the following graph, choose 1, 2, 3 as the tree



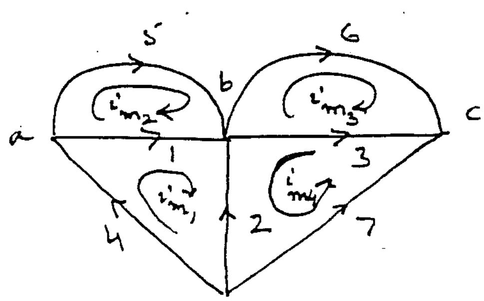
(a) choose appropriate meshes and hence obtain [B]

(b) write [C] ~~an~~

(c) verify that [B] and [C] are orthogonal

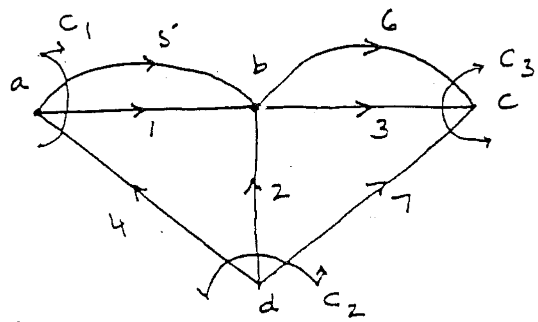
Solution:

(a) The meshes are



$$[B_4] = \begin{array}{c} \\ i_{m_1} \\ i_{m_2} \\ i_{m_3} \\ i_{m_4} \end{array} \begin{array}{c} \begin{array}{ccc:cccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\ \left[\begin{array}{ccc:cccc} 1 & -1 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & -1 & -1 & 0 & 0 & 0 & 1 \end{array}\right] \end{array}$$

(b) The cut-sets are :

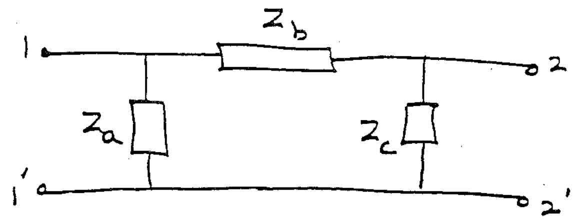


$$[C_3] = \begin{array}{c} \\ c_1 \\ c_2 \\ c_3 \end{array} \begin{array}{c} \begin{array}{ccc:cccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\ \left[\begin{array}{ccc:cccc} 1 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \end{array}\right] \end{array}$$

(c) $[B_4][C_3]^T$

$$= \begin{bmatrix} 1 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & -1 \end{bmatrix} + \begin{bmatrix} -1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} = [\phi].$$

(3) Starting from the definitions, obtain $[z]$ for the π-network shown:



Solution:

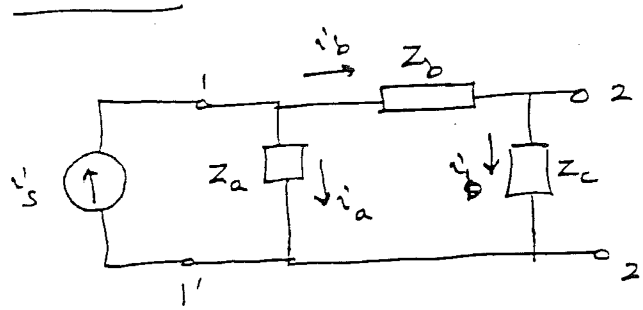
z_{11} = input impedance at 1-1' with 2-2' open-circuited

$$= Z_a \,\|^{l}\, (Z_b + Z_c) = \frac{Z_a(Z_b+Z_c)}{Z_a+Z_b+Z_c}$$

z_{22} = ~~in~~ input impedance at 2-2' with 1-1' open-circuited

$$= Z_c \,\|^{l}\, (Z_a + Z_b) = \frac{Z_c(Z_a+Z_b)}{Z_a+Z_b+Z_c}$$

$z_{12} = z_{12}$: Connect i_s as shown.



$$i_b = \frac{i_s \cdot Z_a}{Z_a+Z_b+Z_c}$$

$$V_{22'} = i_b Z_c$$

This gives $z_{12} = z_{21} = \dfrac{V_{22'}}{i_s} = \dfrac{Z_a Z_c}{Z_a+Z_b+Z_c}$.

1. Fig.1 shows an R-C network, excited by a current source. The various component values are: $R_1 = R_2 = R_3 = 1\ \Omega$, $C = 0.5$ F and $I_s = 1$ A.

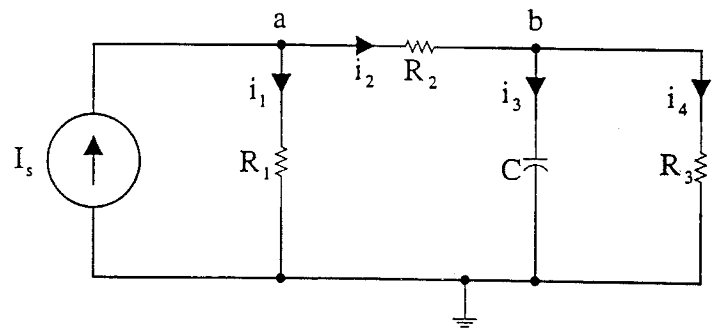


Fig.1.

(a) Taking R_1R_2 to be the tree, write the corresponding [A] matrix.
(b) Obtain the node-admittance matrix $[Y_n] = [A].[Y_B].[A]^T$.
(c) Hence, obtain $v_b(t)$. {Assume $v_b(0) = 0$}.
[Note: $L\{e^{at}\} = \dfrac{1}{s-a}$]

(12 marks)

2. (a) Starting from the definition, obtain the ABCD-parameters of (i) the series impedance Z_s, shown in Fig.2(a), and (ii) the shunt admittance Y_p, shown in Fig.2(b).

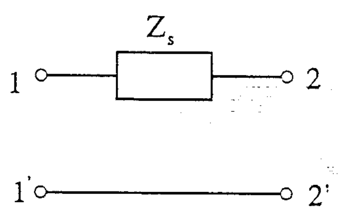


Fig. 2(a)

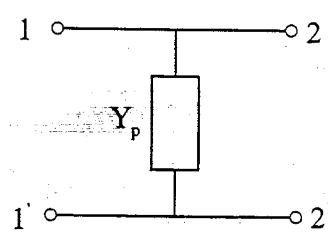


Fig. 2(b)

(6 marks)

(b) Write the electrical equivalent circuit of the mechanical translational system shown in Fig.2(c). (3 marks)

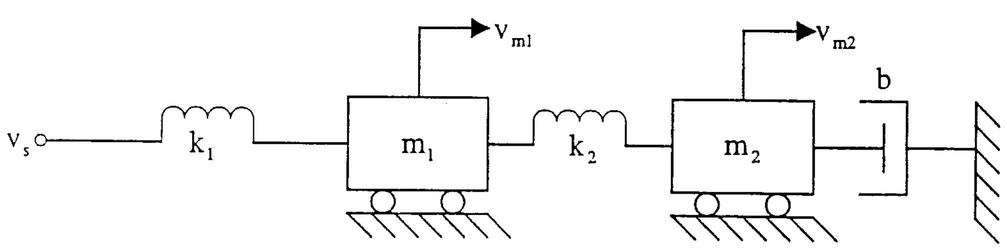


Fig.2(c)

(continued on page 3)

(c) Using the results of Figs.2(a) and (b), obtain the ABCD-parameters of the circuit shown in Fig.2(c). {v_s is the input and v_{m2} is the output) (4 marks)

3. For the network shown in Fig.3,
   (a) determine the y-parameters, starting from the definition.
   (b) Hence, obtain the indefinite admittance matrix.

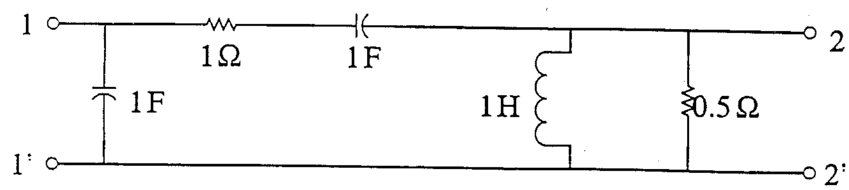


Fig.3.

(8 marks)

4. Fig.4 shows a fluid system.

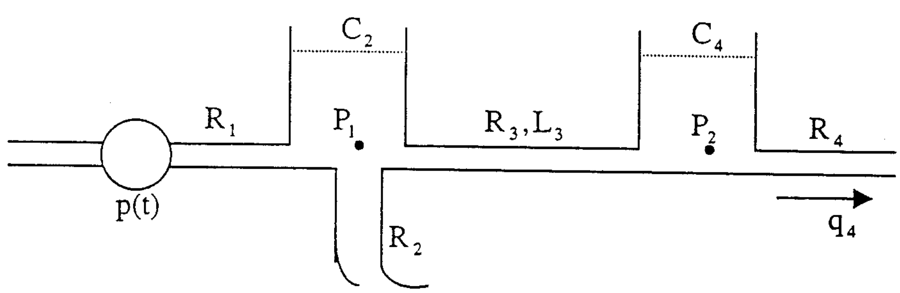


Fig.4

(a) Obtain the analogous electrical equivalent circuit in the Laplace-transform domain.
(b) Write the nodal equations.
(c) Write a signal-flow graph representing these equations.
(d) Hence, obtain $\frac{Q_4(s)}{P(s)}$.

(12 marks)

(continued on page 4)

5. A ladder network is shown in Fig.5.

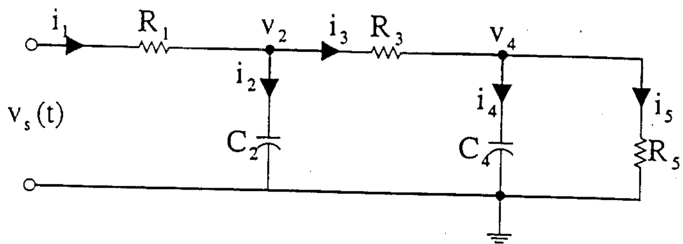


Fig.5.

(a) Obtain the state-equations in the matrix form.
(b) Hence, determine its characteristic equation.
(c) If $C_2 = C_4 = C$ and $R_1 = R_3 = R_5 = R$, show that the characteristic roots are simple and lie on the negative real axis.

(10 marks)

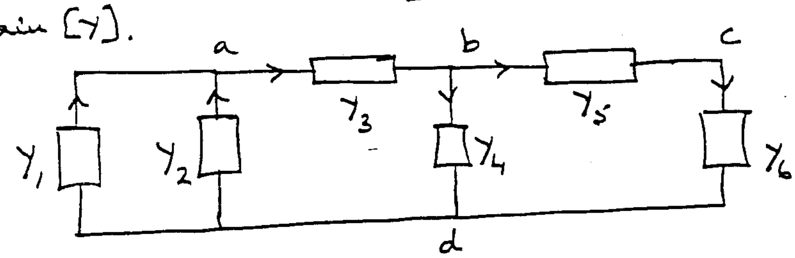
6. (a) Determine the order of the Butterworth low-pass filter which meets the following specifications:
   (i) The attenuation at 1000 Hz is 3 dB.
   (ii) The attenuation at 1500 Hz is greater than 12 dB.
   (b) Obtain the elemental values of a realization, when the terminating resistance is 1000 Ω.

(10 marks)

ENGR 370

Test No. 2

(1) For the network shown below, obtain the node-admittance matrix [Y] in the form $[A_3][Y_B][A_3]^T$. (Take d as the reference node) Hence obtain [Y].



Solution:

$$A_3 = \begin{array}{c} \\ a \\ b \\ c \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} -1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix} \end{array}$$

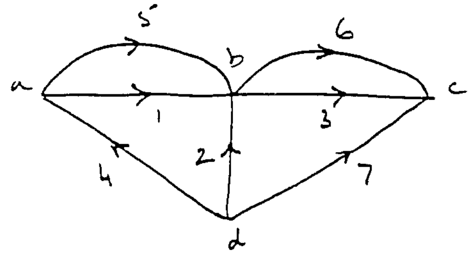
$$[Y_B] = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} Y_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & Y_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & Y_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & Y_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & Y_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & Y_6 \end{bmatrix} \end{array}$$

$$[A_3][Y_B] = \begin{bmatrix} -Y_1 & -Y_2 & Y_3 & 0 & 0 & 0 \\ 0 & 0 & -Y_3 & Y_4 & Y_5 & 0 \\ 0 & 0 & 0 & 0 & -Y_5 & Y_6 \end{bmatrix}$$

$$[A_3][Y_B][A_3]^T = \begin{bmatrix} -Y_1 & -Y_2 & Y_3 & 0 & 0 & 0 \\ 0 & 0 & -Y_3 & Y_4 & Y_5 & 0 \\ 0 & 0 & 0 & 0 & -Y_5 & Y_6 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ -1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (Y_1 + Y_2 + Y_3) & -Y_3 & 0 \\ -Y_3 & (Y_3 + Y_4 + Y_5) & -Y_5 \\ 0 & -Y_5 & Y_5 + Y_6 \end{bmatrix}$$

(2) In the following graph, choose 1, 2, 3 as the tree



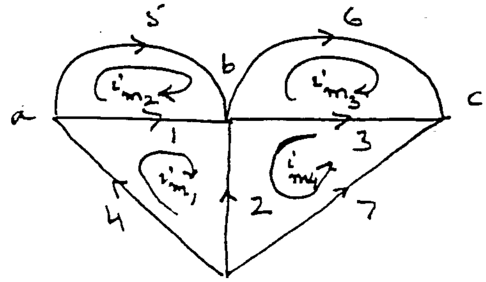
(a) choose appropriate meshes and hence obtain [B]

(b) write [C] ~~an~~

(c) verify that [B] and [C] are orthogonal

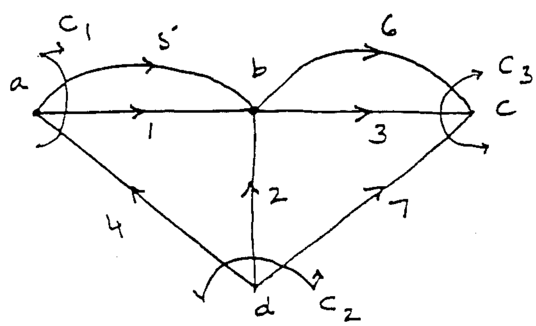
Solution:

(a) The meshes are



$$[B_4] = \begin{array}{c} \\ i_{m_1} \\ i_{m_2} \\ i_{m_3} \\ i_{m_4} \end{array} \begin{array}{c} \begin{array}{ccc|cccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\ \left[\begin{array}{ccc|cccc} 1 & -1 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & -1 & -1 & 0 & 0 & 0 & 1 \end{array}\right] \end{array}$$

(b) The cut-sets are:

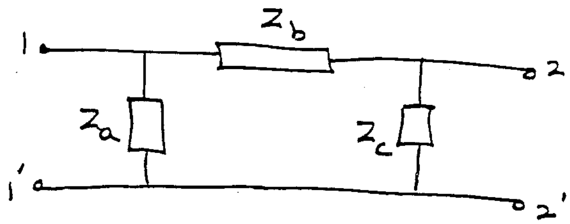


$$[C_3] = \begin{array}{c} \\ c_1 \\ c_2 \\ c_3 \end{array} \begin{array}{c} \begin{array}{ccc|cccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\ \left[\begin{array}{ccc|cccc} 1 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \end{array}\right] \end{array}$$

(c) $[B_4]\,[C_3]^T$

$$= \begin{bmatrix} 1 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & -1 \end{bmatrix} + \begin{bmatrix} -1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} = [\phi].$$

(3) Starting from the definitions, obtain $[z]$ for the π-network shown:



Solution:

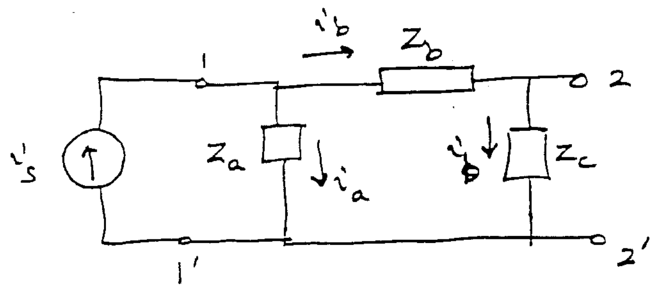
z_{11} = input impedance at 1-1' with 2-2' open-circuited

$$= Z_a \,\|^{l}\, (Z_b + Z_c) = \frac{Z_a(Z_b+Z_c)}{Z_a+Z_b+Z_c}$$

z_{22} = ~~inp~~ input impedance at 2-2' with 1-1' open-circuited

$$= Z_c \,\|^{l}\, (Z_a + Z_b) = \frac{Z_c(Z_a+Z_b)}{Z_a+Z_b+Z_c}$$

$z_{12} = z_{12}$: Connect i_s as shown.



$$i_b = \frac{i_s \cdot Z_a}{Z_a+Z_b+Z_c}$$

$$V_{22'} = i_b Z_c$$

This gives $z_{12} = z_{21} = \dfrac{V_{22'}}{i_s} = \dfrac{Z_a Z_c}{Z_a+Z_b+Z_c}$.

ENGR370 – MODELING AND ANALYSIS OF LINEAR PHYSICAL SYSTEMS
Test No.2 – Winter 2000

Time: One hour

1. Fig.1 shows the graph of a system. Taking the tree to be the branches 1,3,4, obtain the fundamental cut-set matrix.

(4 marks)

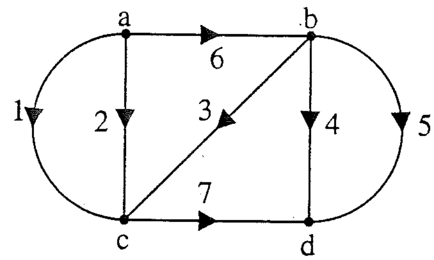


Fig.1.

2. Fig.2 shows a T-network. Starting from the definitions, obtain its y-parameters.

(10 marks)

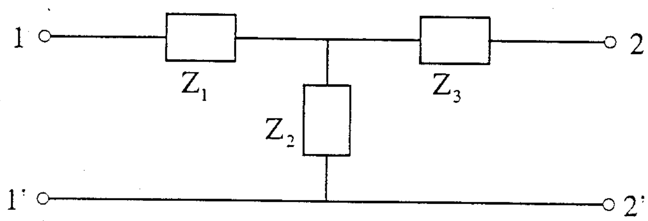


Fig.2.

3. The z-parameters of a network obtained after experimental measurements are given below:

$$[z]=\begin{bmatrix} \frac{2s+1}{s} & \frac{1}{s} \\ \frac{1}{s} & \frac{4s^2+1}{s} \end{bmatrix}$$

Obtain the equivalent T-network. Identify the various components and give their values.

(6 marks)

1. Fig.1 shows an R-C network, excited by a current source. The various component values are: $R_1 = R_2 = R_3 = 1\ \Omega$, $C = 0.5$ F and $I_s = 1$ A.

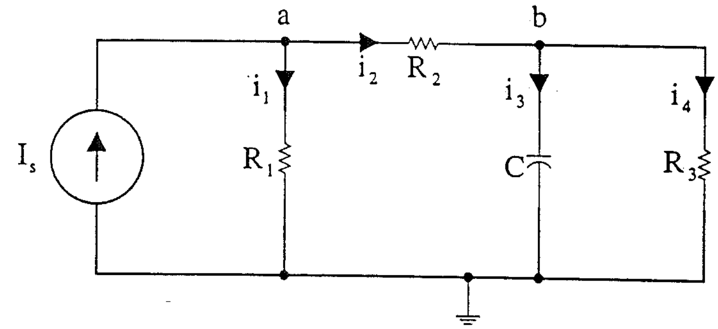


Fig.1.

(a) Taking R_1R_2 to be the tree, write the corresponding [A] matrix.
(b) Obtain the node-admittance matrix $[Y_n] = [A].[Y_B].[A]^T$.
(c) Hence, obtain $v_b(t)$. {Assume $v_b(0) = 0$}.
[Note: $L\{e^{at}\} = \frac{1}{s-a}$]

(12 marks)

2. (a) Starting from the definition, obtain the ABCD-parameters of (i) the series impedance Z_s, shown in Fig.2(a), and (ii) the shunt admittance Y_p, shown in Fig.2(b).

no z param

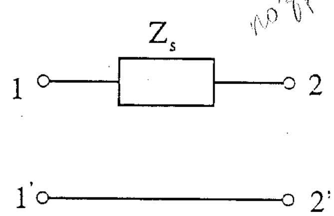


Fig. 2(a)

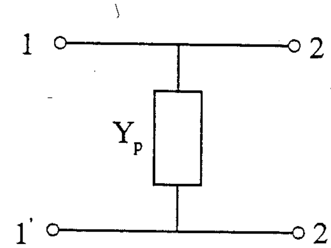


Fig. 2(b)

no y param

(6 marks)

(b) Write the electrical equivalent circuit of the mechanical translational system shown in Fig.2(c). (3 marks)

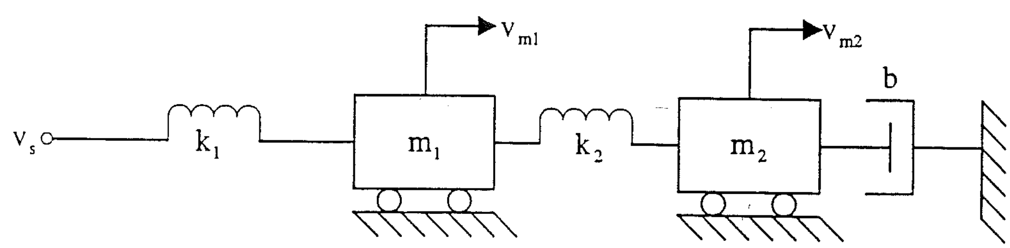
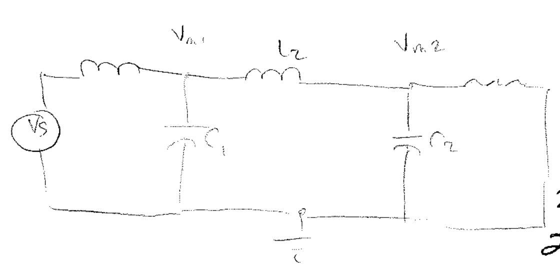


Fig.2(c)



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(c) Using the results of Figs.2(a) and (b), obtain the ABCD-parameters of the circuit shown in Fig.2(c). {v_s is the input and v_{m2} is the output)

(4 marks)

3. For the network shown in Fig.3,
   (a) determine the y-parameters, starting from the definition.
   (b) Hence, obtain the indefinite admittance matrix.

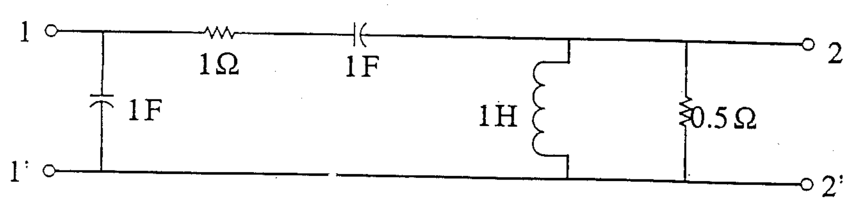

Fig.3.

(8 marks)

4. Fig.4 shows a fluid system.

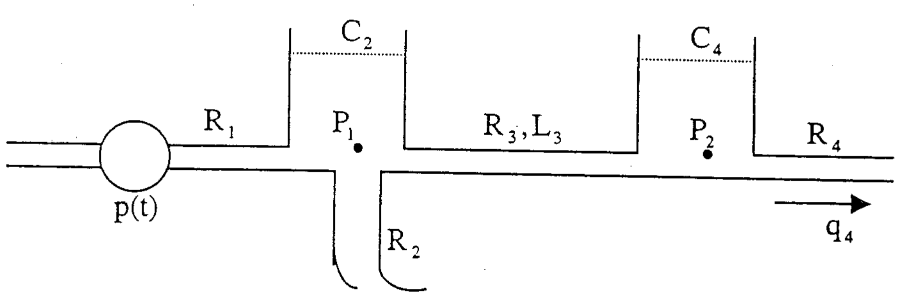

Fig.4

(a) Obtain the analogous electrical equivalent circuit in the Laplace-transform domain.
(b) Write the nodal equations.
(c) Write a signal-flow graph representing these equations.
(d) Hence, obtain $\dfrac{Q_4(s)}{P(s)}$.

(12 marks)

(continued on page 4)

5. A ladder network is shown in Fig.5.

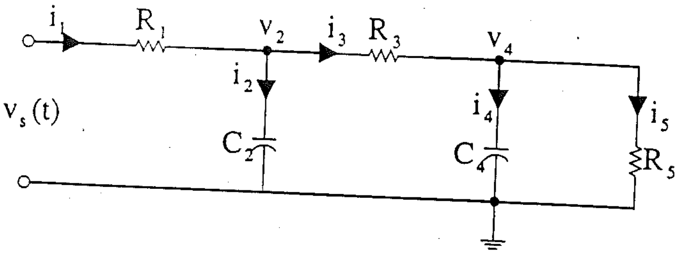

Fig.5.

(a) Obtain the state-equations in the matrix form.
(b) Hence, determine its characteristic equation.
(c) If $C_2 = C_4 = C$ and $R_1 = R_3 = R_5 = R$, show that the characteristic roots are simple and lie on the negative real axis.

(10 marks)

6. (a) Determine the order of the Butterworth low-pass filter which meets the following specifications:
   (i) The attenuation at 1000 Hz is 3 dB.
   (ii) The attenuation at 1500 Hz is greater than 12 dB.
   (b) Obtain the elemental values of a realization, when the terminating resistance is 1000 Ω.

(10 marks)

Concordia
UNIVERSITY

Course	Number	Section
Modeling and Analysis of Linear Physical Systems	ENGR 370	AA

Examination	Date	Time	# of pages
Final Examination	June 2000	3 Hours	5

Instructor(s)

Dr. V. Ramachandran

Materials allowed: [X] No [ ] Yes (Please specify)

Calculators allowed: [ ] No [X] Yes

Students are allowed to use non-programmable electronic calculators without text display.

Special Instructions:

Attempt all questions. Please number and begin each question on a new page.
Show all steps clearly in neat and legible handwriting (preferably in ink).
Students are required to return question paper together with exam booklet(s).

6

Department of Electrical and Computer Engineering
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1. Fig. 1 shows an R – L network, excited by a voltage source. The various component values are: $R_1 = R_2 = R_3 = 1\ \Omega$, $L = 1$ H and $v_s = 1$ volt.

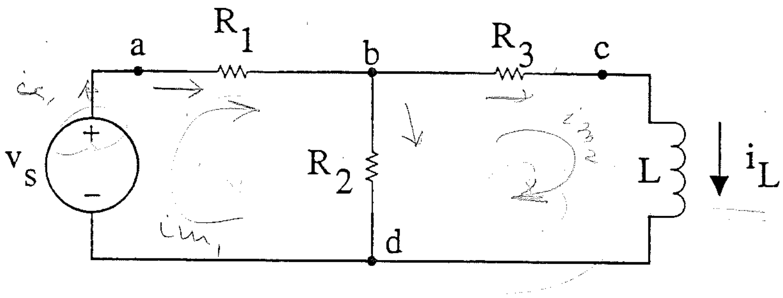

Fig.1

(a) Taking R_1 , R_2 , R_3 to be the tree, write the corresponding matrix [B].
(b) Obtain the mesh-impedance matrix $[Z_m] = [B] . [Z_B] . [B]^T$.
(c) Hence, obtain $i_L(t)$. {Assume $i_L(0) = 0$}

[Note: $L\{e^{at}\} = \frac{1}{s+a}$].

(12 marks)

2. A bridge network is shown in Fig.2.

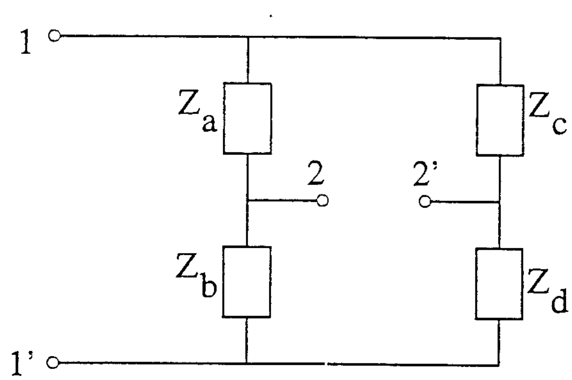

Fig.2

(a) Starting from the definitions, obtain its y-parameters.
(b) Given $[y] = [z]^{-1}$, obtain its z-parameters.

(10 marks)

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3. For the network shown in Fig.3,

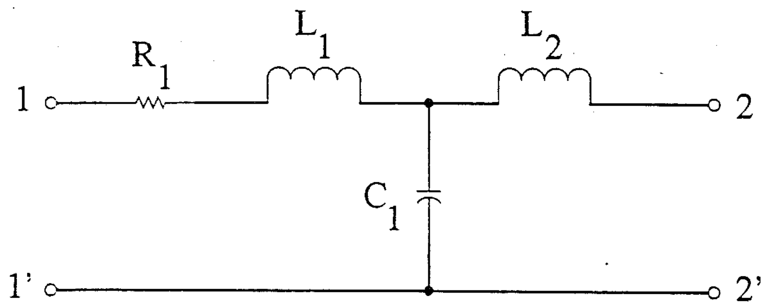


Fig.3

(a) Determine the z-parameters, starting from the definitions.
(b) Obtain its y-parameters.
(c) Hence, obtain the indefinite admittance matrix.

(11 marks)

4. Fig.4 shows a thermal system.

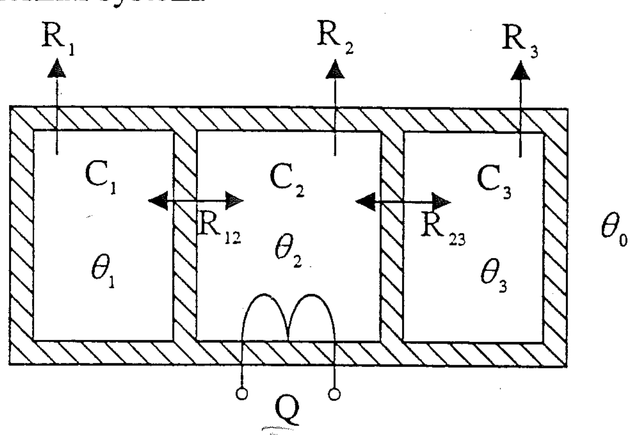


Fig.4

(a) Obtain the analogous electrical equivalent circuit in the Laplace-Transform domain.
(b) Write the nodal equations.
(c) Write a signal-flow graph representing these equations.
(d) Hence, obtain $\theta_1(s)$, $\theta_2(s)$, and $\theta_3(s)$.

(12 marks)

5. Fig. 5 shows a mechanical system.

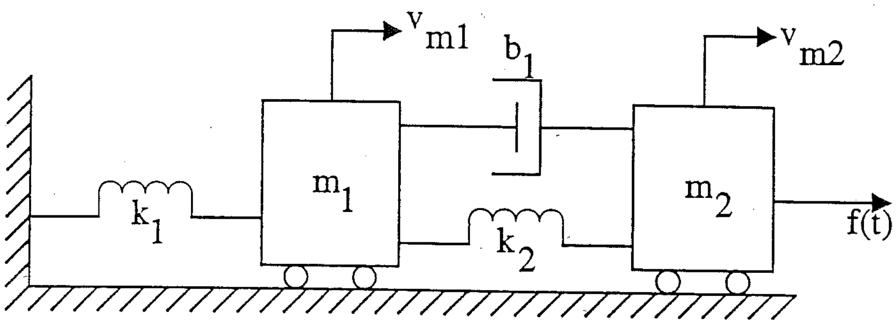


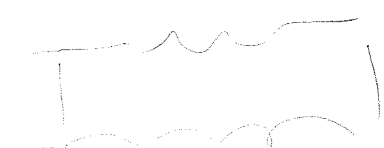
Fig.5

(a) Obtain its electrical equivalent circuit.
(b) Write the state-equations in the matrix form.
(c) Hence, determine its characteristic equation.

(10 marks)

6. (a) Determine the order of the Butterworth low-pass filter which meets the following specifications:
   (i) The attenuation at $\omega_c = 4000$ radians/second is 3 dB.
   (ii) The attenuation at $\omega_a = 6000$ radians/second is 14 dB.
   (b) Obtain the elemental values of a realization, when the terminating resistance is 75 ohms.

(10 marks)



1. Fig.1 shows the graph of a network.

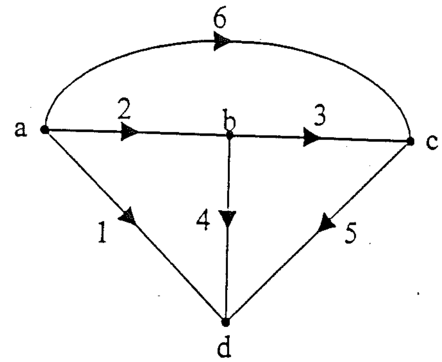


Fig.1.

Choose the branches 1,2 and 3 as the tree and d as the reference node.
(a) Write the node-branch incidence matrix $[A_3]$.
(b) Write the mesh-branch incidence matrix $[B_3]$. (Choose appropriate meshes)
(c) Write the fundamental cut-set matrix $[C_3]$. (Choose appropriate cutsets)
(d) Compute $[A_3].[B_3]^T$.
(e) Compute $[B_3].[C_3]^T$.

(13 marks)

2. Fig.2 shows a mechanical system.

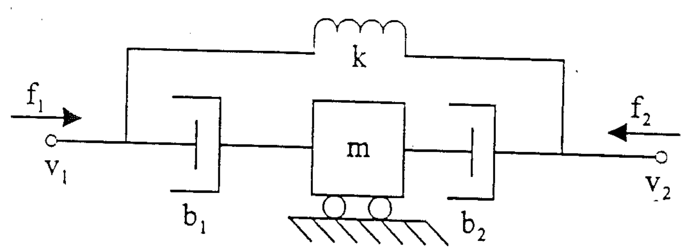


Fig.2.

This can be considered as a two-port network.
(a) Obtain the corresponding electrical equivalent network in the Laplace-transform domain.
(b) Determine its [z] parameters.
(c) Determine its [y] parameters.

(12 marks)

10

3. Fig.3 shows a model of a field-effect transistor (FET).

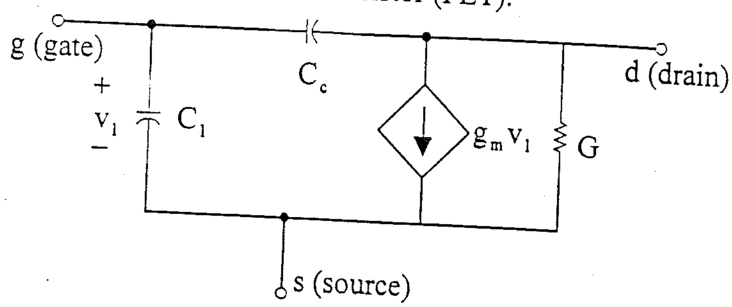


Fig.3

(a) Obtain the admittance matrix, when node s is taken as the reference.
(b) Obtain its indefinite admittance matrix (IAM)
(c) Hence, obtain the admittance matrix, when g is taken as the reference.

(10 marks)

4. Fig.4 shows a fluid system.

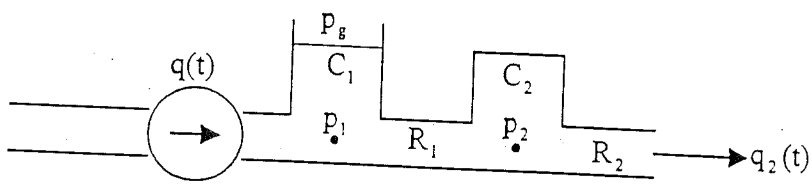


Fig.4.

(a) Obtain the equivalent electrical system in the Laplace transform domain.
(b) Write the nodal equations.
(c) Write a signal-flow graph corresponding to these equations.
(d) Hence, determine $\dfrac{Q_2}{Q}$ as a function of s.

(10 marks)

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5. Fig.5 shows a thermal system.

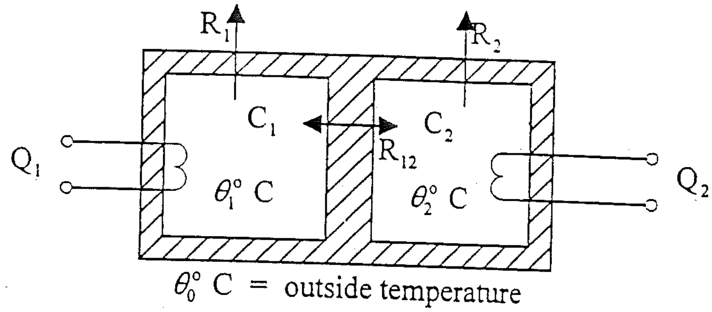

Fig.5.

(a) Write the analogous electrical equivalent circuit.
(b) Write the state equations of the system in the matrix form.
(c) Assume $C_1 = C_2 = C$, $R_1 = R_2 = R_{12}$. Show that the characteristic equation $|sI - A|$ has distinct, negative-real axis roots. { I is the unit matrix}.

(10 marks)

6. It is required to obtain a Butterworth low-pass filter which should have an attenuation of at least 12 dB at $\Omega = 1.5$ radians/second.
(a) Determine the minimum order of the transfer function required.
(b) For this transfer function, obtain a realization.
(The table of Butterworth transfer function is enclosed herewith).

(10 marks)

12

1.92

Concordia
UNIVERSITY

Course	Number	Section
Modeling and Analysis of Linear Physical Systems	ENGR 370	All Sections

Examination	Date	Time	# of pages
Final Examination	December 2000	3 Hours	5

Instructor(s)

Professors D. Davis and V. Ramachandran (course coordinator)

Materials allowed: [X] No [ ] Yes (Please specify)

Calculators allowed: [ ] No [X] Yes

Students are allowed to use ONLY non-programmable calculators WITHOUT text display.

Special Instructions:

Attempt all six (6) questions. Please number and begin each question on a new page.
Students are required to return question paper together with exam booklet(s).

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1. Fig.1 shows the directed graph of a network. Taking 2,4,6 as the tree,
   (a) Write [B].
   (b) Write [C].
   (c) verify $[B].[C]^T = 0$.
   (d) Hence, obtain the total number of trees that exist for this graph.
   (e) List all the trees.

(12 marks)

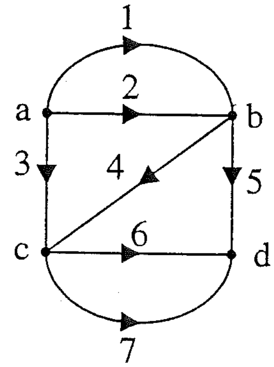

Fig.1.

2. Fig.2 shows two two-port networks N_a and N_b connected in series. The z-parameters are given as:

$$[z]N_a = \begin{bmatrix} 50 & 25 \\ 25 & 100 \end{bmatrix} \text{ and } [z]N_b = \begin{bmatrix} 50 & 25 \\ 25 & 100 \end{bmatrix}$$

(a) Determine $[z]_{Total}$. $[Z_a] + [Z_b]$
(b) If $I_1 = 3$ mA and $I_2 = 4$ mA, determine V_{Total} at Port 1-1′ V_1 $\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = [Z_{Total}]\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$
(c) What is the input power? $P_I = V_I \cdot I_I$

(11 marks)

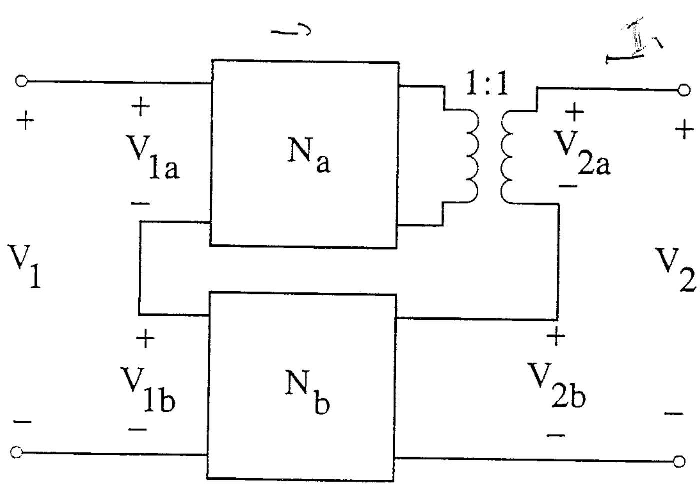

Fig.2.

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3. Fig.3 shows the passive portion of an electrical network.
   (a) Obtain [A] without taking any node as the reference.
   (b) Write the branch-admittance matrix $[Y_B]$.
   (c) Hence, determine its indefinite admittance matrix given by $[A].[Y_B].[A]^T$.

(11 marks)

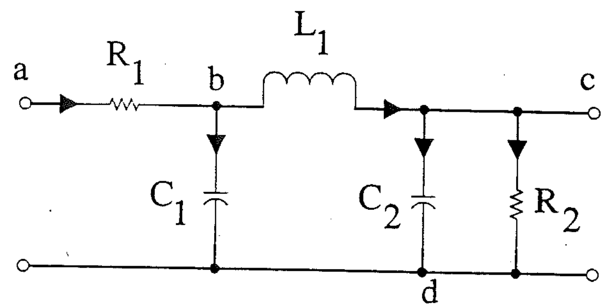


Fig.3.

4. Fig.4 shows a mechanical (rotational) system.
   (a) Obtain its equivalent electrical network.
   (b) Write the set of nodal equations.
   (c) Write a signal-flow graph corresponding to the nodal equations.
   (d) Hence, determine $\frac{\omega_2}{T(s)}$.

(12 marks)

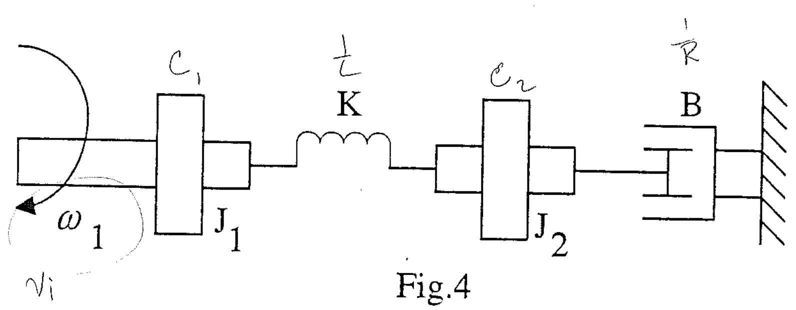


Fig.4

..4/-

5. Fig.5 shows a fluid system.
   (a) Obtain its electrical equivalent network.
   (b) Write the state-variable equations.
   (c) Using the state-equations, determine the conditions under which the system is (i) over-damped, (ii) critically damped, and (iii) under-damped.

(12 marks)

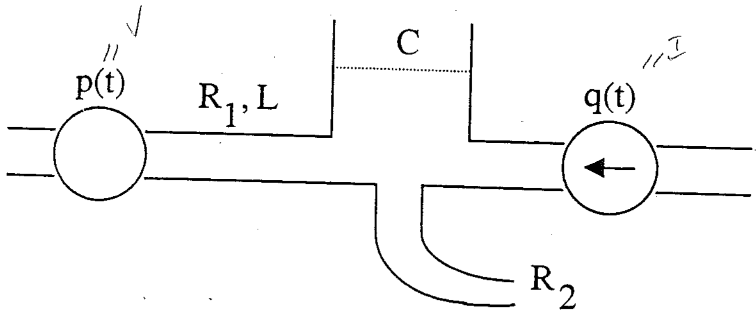


Fig.5.

6. For a medical device, we need a filter that will attenuate all input signals by at least 20 dB that have a frequency of 60 Hz or higher. Design a Butterworth filter to accomplish this, if the passband is 20 Hz and the terminating resistance is 75 ohms.

(12 marks)

..5/-

Table

Coefficients of the various Butterworth Polynomials upto order n = 8.

$$D_B(S) = a_n S^n + a_{n-1} S^{n-1} + \ldots + a_{n-k} S^{n-k} + \ldots + a_2 S^2 + a_1 S + a_0$$

Also, $a_{n-i} = a_i$

Order	
n=1	$a_1 = a_0 = 1$
n=2	$a_2 = a_0 = 1$ $a_1 = 1.414214$
n=3	$a_3 = a_0 = 1$ $a_2 = a_1 = 2$
n=4	$a_4 = a_0 = 1$ $a_3 = a_1 = 2.613216$ $a_2 = 3.414214$
n=5	$a_5 = a_0 = 1$ $a_4 = a_1 = 3.236068$ $a_3 = a_2 = 5.236068$
n=6	$a_6 = a_0 = 1$ $a_5 = a_1 = 3.863703$ $a_4 = a_2 = 7.464102$ $a_3 = 9.141620$
n=7	$a_7 = a_0 = 1$ $a_6 = a_1 = 4.493959$ $a_5 = a_2 = 10.097835$ $a_4 = a_3 = 14.591794$
n=8	$a_8 = a_0 = 1$ $a_7 = a_1 = 5.125831$ $a_6 = a_3 = 21.846151$ $a_5 = a_3 = 21.846151$ $a_4 = 25.688356$

ENGR370 - MODELING AND ANALYSIS OF LINEAR PHYSICAL SYSTEMS
Test No.1 - Summer 1999

Time: One hour

1. List all the trees of the graph shown in Fig.1.

(7 marks)

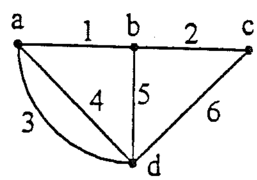


Fig.1.

2. Identify the following components and obtain their magnitudes and units. Give the analogous electrical component. Give reasons for your answer.

(6 marks)

(a) The deflections of a cantilever beam are found as below:

f, N	Deflection, cm
0	0
1	1.5
2	3.0
3	4.5

(b) A fluid is flowing into a container at a constant rate of 1.5 cm^3/second. The pressure in the container is found as follows:

Time, seconds	Presssure, N/cm^2
0	0
1	3
2	6
3	9

3. Fig.3 shows a fluid system.

(7 marks)

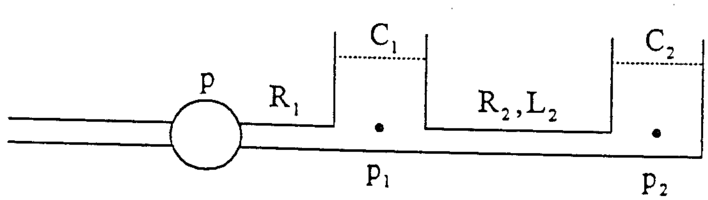


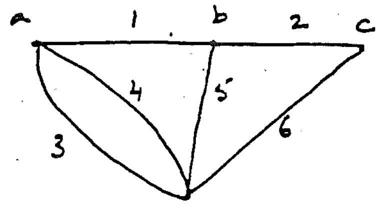
Fig.3.

(a) Obtain the analogous electrical system.
(b) Obtain the equivalent circuit in the Laplace transform domain.
Give reasons for your answer.

ENGR 370 Summer 1999

Test No. 1

1) List all the trees of the graph shown below



Solution

The trees are: 123, 124, 125, 126, 136, 146, 156, [redacted] 235, 236, 245, 246, 356, 456.

2) Identify the following components and obtain their magnitude and units. Give the analogous electrical component. Give reasons for your answer.

(a) The deflections of a cantilever beam are found as below:

F, N	Deflection
0	0
1	1.5 cm
2	3 cm
3	4.5 cm

Solution: $F = Kx$. K is a spring constant $= \frac{2}{3}$ N/cm

$= \frac{2}{3}(10^{+2})$ N/metre

The analogous element is an inductor $L = \frac{1}{K}$.

(b) A fluid is flowing into a container at a constant rate of 1.5 cm^3/second. The pressure in the container is found as follows:

Time, Second	Pressure, N/cm^2
0	0
1	3
2	6
3	9

Solution: $c = \dfrac{q}{\frac{dP}{dt}} = \dfrac{(1.5)(10^{-6})}{(3)(10^{-4})} = (0.5)\,10^{-10}\ m^5/N.$

This is a fluid capacitance. The analogous element is an electrical capacitance.

3) Fig. 3 shows a fluid system.

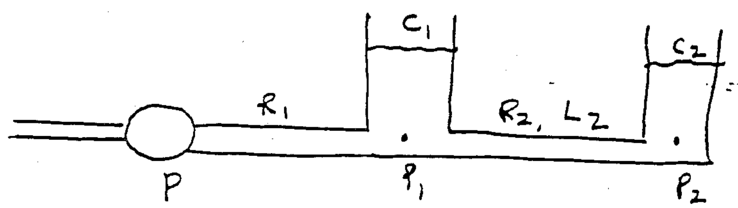
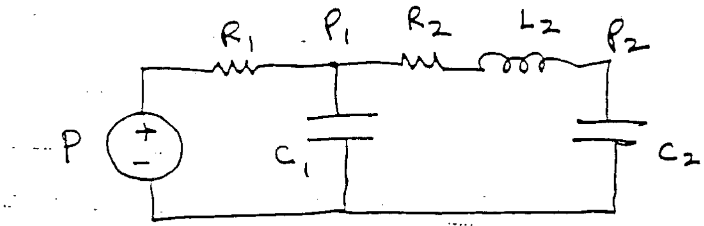


Fig. 3

a) obtain the analogous electrical system
b) obtain the equivalent circuit in the Laplace transform domai
Give reasons for your answer

Solution: (a)



(b)