

Engineering Differential Equations
Section J
Exam I (A)

Directions: You have **60 minutes** to solve the following **4** problems. You may use a calculator. No cell phones are allowed during the exam.

- (1) (10 points) For each of the following first-order differential equations, state whether it is separable, linear, homogeneous, or none of the above. You do not need to explain your answers. Each equation may correspond to more than one listed type (in which case you must list **all** types) or none of those.

(a) $\frac{dy}{dx} = x - y$

(b) $\frac{dy}{dx} = \sqrt{x - y}$

(c) $\frac{dy}{dx} = y\sqrt{x}$

(d) $\sqrt{xy} dy - y dx = 0$

(e) $\sqrt{xy} \frac{dy}{dx} = \sqrt{x - y}$

- (2) (10 points) Find a general solution of the first-order linear equation

$$y' = 2y + e^{-2x}.$$

- (3) Let

$$\left(\frac{2x}{x^2 + y^2 + 1} + y \right) dx + \left(\frac{2y}{x^2 + y^2 + 1} + x \right) dy = 0.$$

- (a) (5 points) Check that this is an exact differential equation.
(b) (5 points) Solve the equation leaving the general solution in implicit form.

- (4) A rabbit population P satisfies the growth equation

$$\frac{dP}{dt} = kP, \quad k > 0.$$

- (a) (5 points) Find the general solution of this differential equation.
(b) (5 points) Suppose that at time $t = 0$ (months) the population numbers 100 rabbits and is increasing at the rate of 20 rabbits per month. How many rabbits will be there one year later? (Give the exact value calculated with this model, do not approximate.)