SOME 275 FINALS (WITH SOLNS) (23 pages)

(1)

INSTR: N. SURESH H 881.02 848-2424, Local 3157

1. Using **mesh analysis**, find the voltage 'v' in the circuit of Fig.1. (Use the mesh currents shown).

(8 marks)

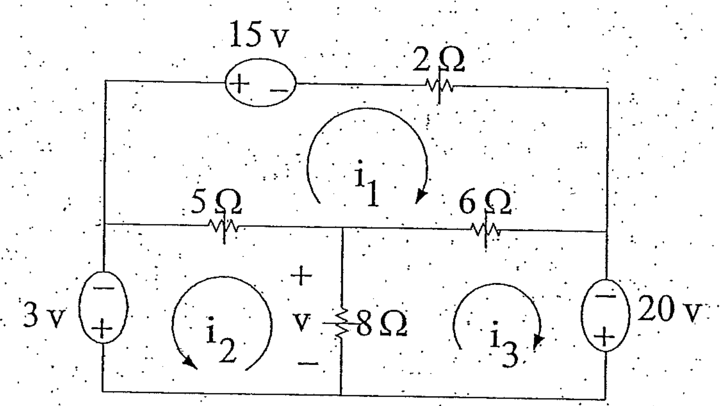

Fig.1

2. For the circuit shown in Fig.2,
   (a) Obtain the Thevenin equivalent circuit across a-b, after disconnecting R_L. (Hint: use Superposition theorem).
   (b) Find the value of the load resistance R_L such that maximum power is delivered to it.
   (c) What is the maximum power delivered ?

(7 marks)

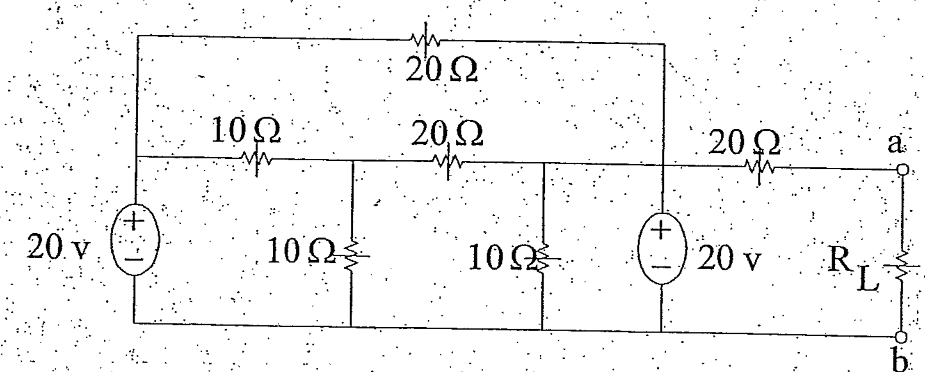

Fig.2

..3/-

3. An operational amplifier circuit is shown in Fig.3. Find
   (a) V_{out},
   (b) the currents i_1, i_2 and i_3,
and (c) the voltage 'v' across R_3.
The various component values are: $R_1 = R_2 = 2000$ ohms, $R_3 = 350$ ohms, $R_4 = 650$ ohms, $R_5 = 1000$ ohms and $v_s = 2$ volts.

(6 marks)

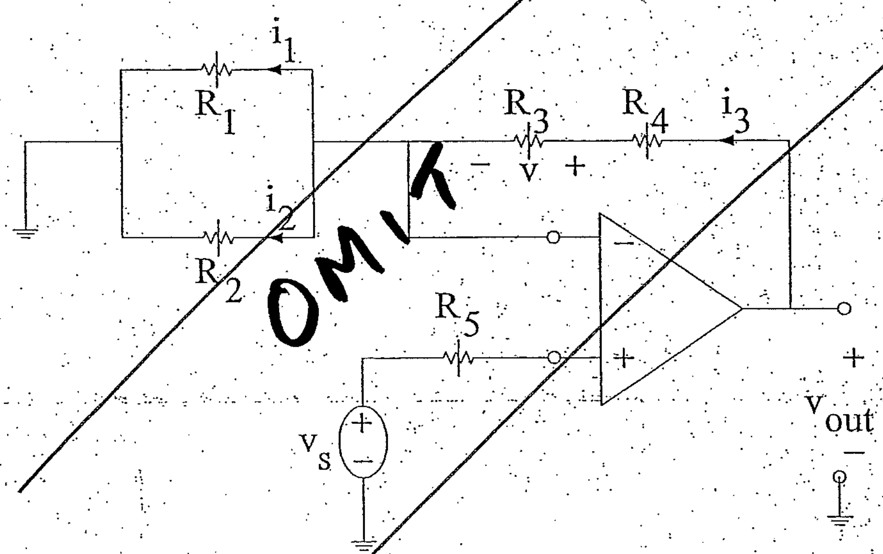

Fig.3

4. The switch S of Fig.4 has been in position B for a long time. The switch is placed in position A at $t = 0$. Find $i(t)$ for $t \geq 0$.

(8 marks)

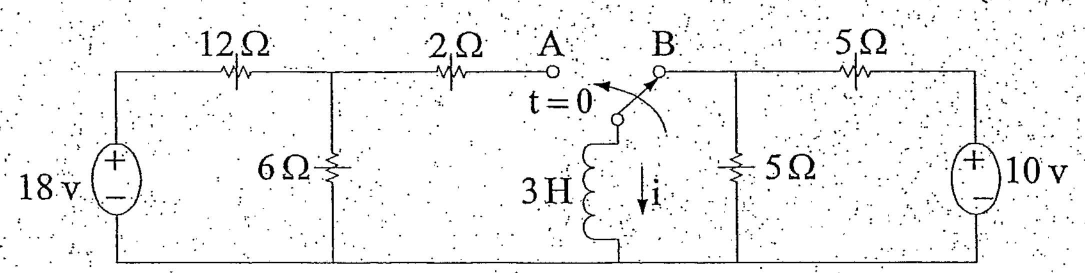

Fig.4

..4/-

INSTR: N. SURESH H 881.02
848-2424, local 3157

1. Using **mesh analysis**, find the voltage ‘v’ in the circuit of Fig.1. (Use the mesh currents shown).

(8 marks)

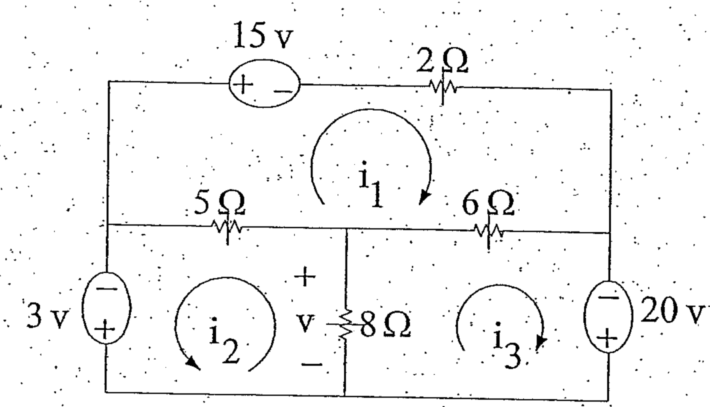

Fig.1

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   (a) Obtain the Thevenin equivalent circuit across a-b, after disconnecting R_L. (Hint: use Superposition theorem).
   (b) Find the value of the load resistance R_L such that maximum power is delivered to it.
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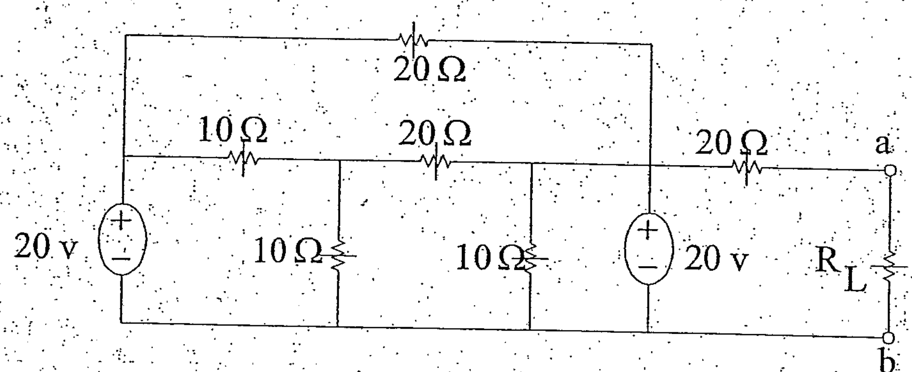

Fig.2

..3/-

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   (b) the currents i_1, i_2 and i_3,
and (c) the voltage 'v' across R_3.
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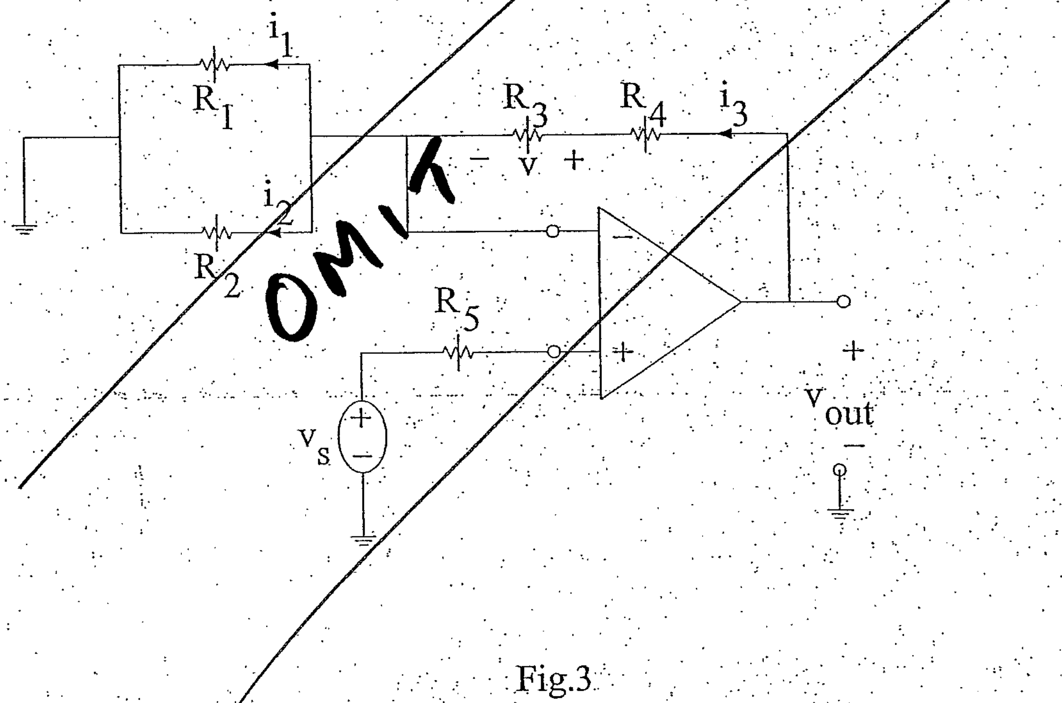


Fig.3

4. The switch S of Fig.4 has been in position B for a long time. The switch is placed in position A at t = 0. Find i(t) for t ≥ 0.

(8 marks)

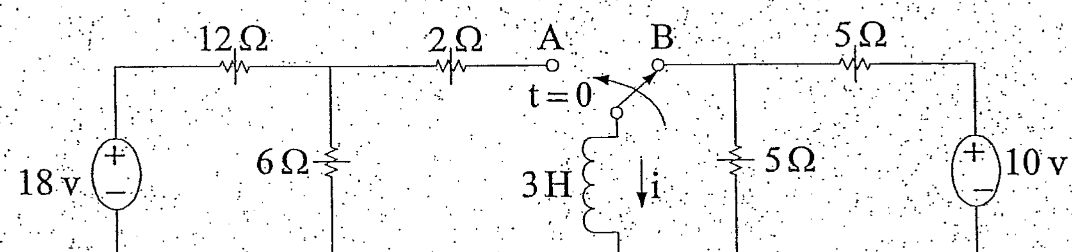


Fig.4

..4/-

5. The circuit shown in Fig.5 has zero stored energy at $t = 0$. Determine $i_L(t)$ for $t \geq 0$. The various component values are : $V = 6$ volts, $L = 3$ H, $C = \frac{1}{9}$F, $R = 6\sqrt{3}$ ohms.

(8 marks)

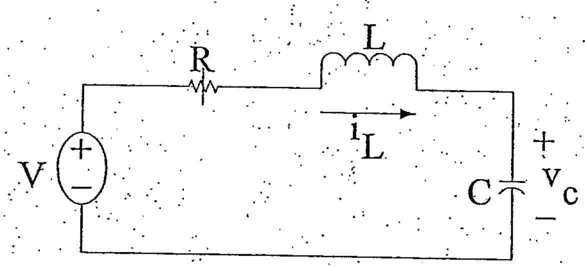


Fig.5

6. For the circuit shown in Fig.6,
   (a) Find the value of C that will make the impedance seen by the voltage source purely resistive.
   (b) Find the phasor current I, if $C = 12\ \mu$F.
   (c) Find the rms real power delivered by the source under condition (b).
   The voltage source is 64.5 v (rms) at a frequency of 4000 radians/second.

(8 marks)

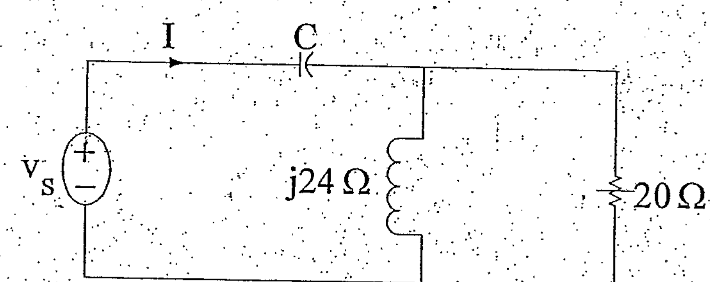


Fig.6

..5/-

(4)

7. The circuit of Fig.7 shows an impedance matching unit.
   (a) Determine the values of 'n' and L so that maximum power transfer to R_2 is ensured.
   (b) Hence, determine the maximum power absorbed by R_2.

The various component values are: $R_1 = 600$ ohms, $C = 10^{-7}$ F, $R_2 = 2400$ ohms and $v_s = 10$ v (rms) at a frequency of 1000 Hz.

(8 marks)

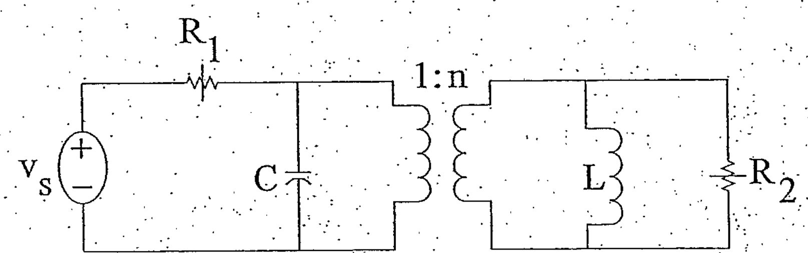


Fig.7

8. Fig.8 shows a three-phase power supply unit. Three loads A, B and C are being supplied power as indicated by a balanced 3-phase source with a line voltage of 208 volts (RMS) at 60 Hz. The various specifications of the loads are as follows:

Load A: $P_{rms} = 25$ kW at 0.65 lagging power factor;
Load B: $P_{rms} = 15$ kw at unity power factor,
Load C: $P_{app} = 10$ kVA at 0.8 leading power factor.

   (a) Determine the complex powers in each load in the form $S = P + jQ$ watts.
   (b) The sum of the complex powers determines the magnitude of the total line current magnitude I_L. Find I_L.
   (c) Determine the power factor of the combined load.

(12 marks)

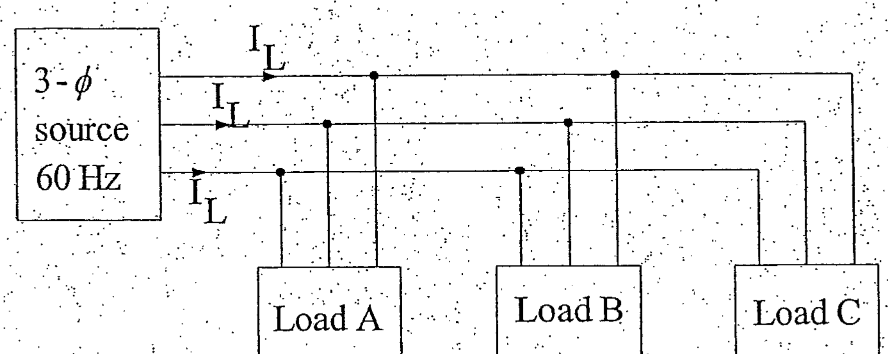
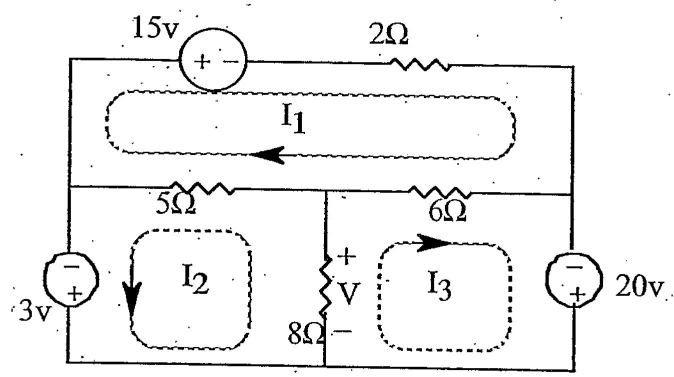


Fig.8

5

FINAL Dec 2005 SOLUTIONS

Q1: Voltage V by Mesh Analysis



Solution:

$V = -8\,(I_2 + I_3)$

Mesh Eqns :

$2I_1 + 6(I_1 - I_3) + 5(I_1 + I_2) + 15 = 0$ or $13I_1 + 5I_2 - 6I_3 = -15$

$8(I_2 + I_3) + 5(I_1 + I_2) - 3 = 0$ or $5I_1 + 13I_2 + 8I_3 = 3$

$8(I_3 + I_2) + 6(I_3 - I_1) - 20 = 0$ or $-6I_1 + 8I_2 + 14I_3 = 20$

$$I_2 = \frac{\begin{vmatrix} 13 & -15 & -6 \\ 5 & 3 & 8 \\ -6 & 20 & 14 \end{vmatrix}}{\begin{vmatrix} 13 & 5 & -6 \\ 5 & 13 & 8 \\ -6 & 8 & 14 \end{vmatrix}} = \frac{-1534 + 1770 - 708}{1534 - 590 - 708} = -472/236 = -2\text{ A}$$

$$I_3 = \frac{3068 - 590 - 1770}{236} = 708/236 = 3\text{ A}$$

$V = -8\,(-2+3) = \underline{-\mathbf{8}\ \text{volts}}$

(6)

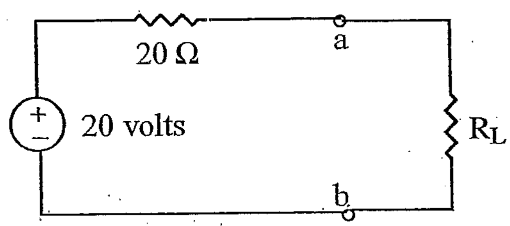
Q2 : Determine Thev Eq Cct across ab, R_L for max power transfer., and P_L (max)

Solution: Superposition yields V_{ab} (oc) = V_{ab1} + V_{ab2} = 0 + 20 = 20 volts

∴ V_{Th} = V_{ab} (oc)= **20 volts** .

Also , Superposition yields I_{ab} (sc) = I_{ab1} + I_{ab2} = 0 + 20/20 = 1 A

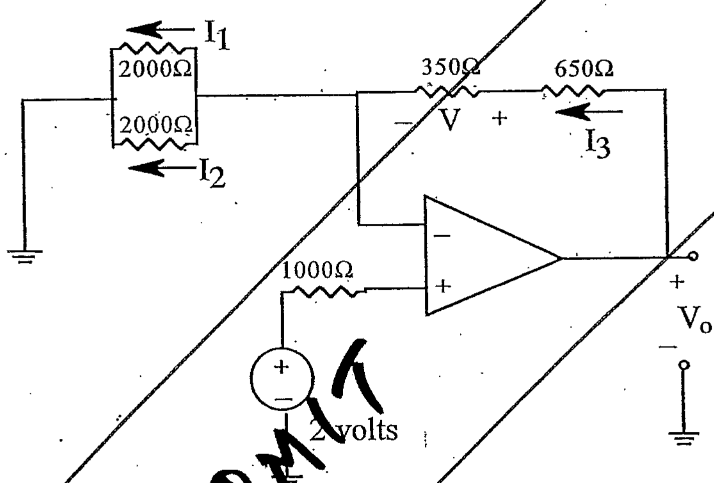
∴ R_{Th} = V_{ab} (oc)/ I_{ab} (sc) = 20/1 = **20Ω** volts .



(b) For MPT R_L = R_{Th} = **20Ω**

(c) P_L (max) = $(V_{Th})^2 / 4\ R_{Th}$ = 400/80 = 5 Watts

Q3: Determine voltage V_o, currents I_1 , I_2 , I_3 and voltage V. .



OMIT

Solution: Circuit is a "non-inverting amplifier"

V_0 = [1 + (650+350)/ (2000P2000)](2) = [1 + 1] (2) = 4 volts

Current I_3 = [V_0 − 2]/1000 = 2/1000 A = 2 mA

Currents I_1 = I_2 =[2]/2000 = 2/2000 A = 1 mA

Voltage V = 350 I_3 = 350 (2) mV = **0,7 volts** ie (**700** mV)

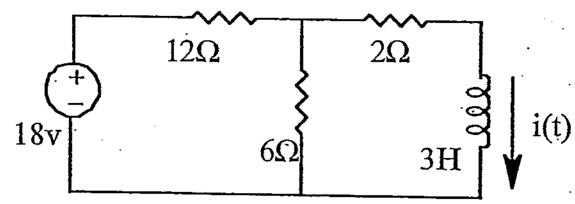
(7)

Q4 : Determine $i(t)$, $t>0$

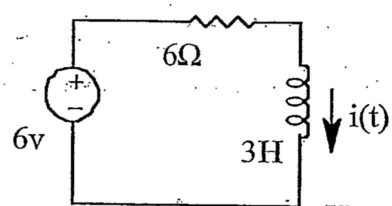
Solution:

$i(0-) = 10/5 = 2\ A = i(0+)$.

For $t>0$, the circuit is



Reducing the circuit connected to L to a Thevenin Eq Cct, we have
$V_{Th} = (18)(6/18) = 6$ volts and $R_{Th} = 2 + 12P6 = 2 + 4 = 6\Omega$



By differential eqn., $-6 + 6i + 3\ di/dt = 0$
or $0.5\ di/dt + i = 1$
from which, $i(t) = Ae^{-2t} + 1$
and $A = i(0+) - 1 = 2 - 1 = 1$

Hence

$$i(t) = \mathbf{1}e^{-2t} + \mathbf{1} \quad \text{,Amp}$$

{ If a 'circuit approach' is used, the time-constant $\tau = L/R_{eq} = 3/6 = 0.5$ sec and $i_{ss}(t) = 6/6 = 1$ A. Hence $i(t) = Ae^{-2t} + 1$ and $A = i(0+) - 1 = 2 - 1 = 1$ and the result is $i(t) = \mathbf{1}e^{-2t} + \mathbf{1}$,Amp }

Q5: Determine $i_L(t)$, $t>0$

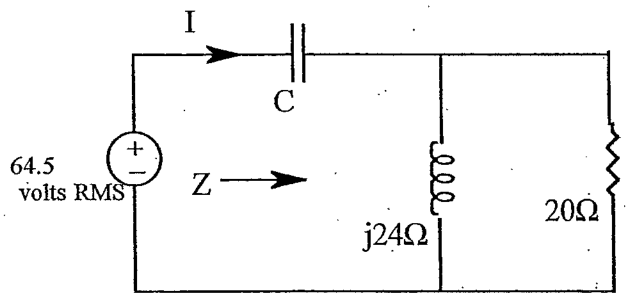
Solution: The circuit is a "series RLC" with $\alpha = R/2L = 6\sqrt{3}/6 = \sqrt{3}$ and $\omega_n = 1/\sqrt{(LC)} = 1/\sqrt{(3/9)} = \sqrt{3} = \alpha$. Hence the circuit is critically- damped and $i_L(t) = [A_1 + A_2 t]e^{-\sqrt{3}t} + i_{Lss}(t)$

In the steadystate, $L \rightarrow$ oc and $i_{Lss}(t) = 0$, hence $i_L(t) = [A_1 + A_2 t]e^{-\sqrt{3}t}$.
and $di(t)/dt = -[A_1 + A_2 t]\sqrt{3}\,e^{-\sqrt{3}t} + e^{-\sqrt{3}t} A_2$.
Now, $i(0-) = 0 = i(0+)$ $\quad \therefore [A_1 + 0]1 = 0$ or $A_1 = 0$.

$di(0+)/dt = V_L(0+)/L = 6/3 = 2$ A/s, $\therefore -\sqrt{3}A_1 + A_2 = 2$ or $A_2 = 2$

Therefore $i_L(t) = \underline{2\ t\ e}^{-\sqrt{3}t}$. Amp

Q6: (a) Determine C which will make Z purely resistive, (b) the phasor current I if C=12μF, (c)rms power delivered by the source under conditions of (b).



Solution: $\omega = 4000$ rad/s

(a) $Z = -1/j\omega C + \{20(j24)\}/(20+j24) = -1/j\omega C + j480\,(20-j24)/(400+576)$

$= -1/j4000C + (j9.836 + 11.8) = 11.8 + j(9.836 - 1/4000C)$

Setting Im Z = 0, $(9.836 - 1/4000C) = 0$ and $C \approx$ **25.4** μF

(b) If C = 12μF, $Z = 11.8 + j(9.836 - 1/4000C) = 11.8 - j10.997 = 16.13\angle{-43^\circ}$

$I = 64.5\angle 0^\circ / 16.133\angle{-43^\circ} \approx$ **4** $\angle 43^\circ$ A

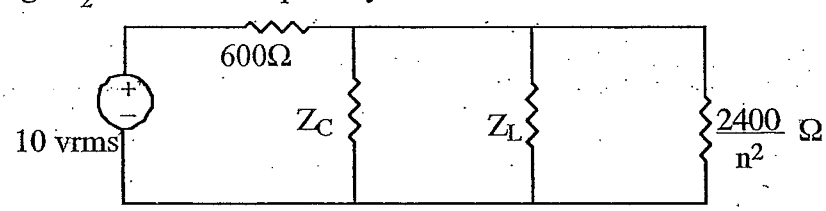
(c) Power $P = VI\cos\theta$ where θ is the total impedance-angle $= 0-43 = \angle{-43^\circ}$

Thus $P = 64.5\,(16.13)\cos(-43^\circ) = 258(0.731) =$ **188.6** Watts

Q7 (a) Determine (a) n and L for maximum power transfer to R_2., (b) the maximum power absorbed by R_2.

Solution: $\omega = 2\pi.\,1000 = 6280$ rad/s

Referring R_2. and L to the primary side we have



ie $Y_C = 1/Z_C = j\,(6280)10^{-7}$

$Z_L = j\,(6280)L/n^2$ or $Y_L = -jn^2/(6280)L$

(a) For MPT, the two requirements are,

$Y_L = -Y_C$ or $n^2/(6280)L = (6280)10^{-7}$

and $2400/n^2 = 600$. Thus, $n = \sqrt{(2400/600)} = \sqrt{4} =$ **2**

and $L = 4/(6280)^2 10^{-7} = 4/3.944 \approx$ **1.01** H

(b) With the MPT conditions satified, $P_{R2}(\max) = (10)^2/4(600) =$ **41.7** mW

Q8 Power is being supplied to three 3-phase loads A: P_{rms} = 25kW @ 0.65pf lagging, B : P_{rms}=15 kW @ unitypf, and C: P_{app}=10 kVA @ 0.8 leading pf. The line voltage is 208 Vrms at 60 Hz. (a) Determine the complex power in each load in the form S=P+jQ Watts., (b) The total line current I_L is determined by the sum of the complex powers. Find I_L . (c) Determine pf of the combined load.

Solution: $S = P\,[\,1 + j\tan\theta\,]$ where $\theta = \cos^{-1}pf$

Thus for loads A and B , we have

$$S_A = 25000\,(1 + j\tan 49.6^\circ) = \underline{\mathbf{(25000 + j29228.24)}}\ \text{Watts}$$

and

$$S_B = 15000\,(1 + j\tan 0^\circ) = \underline{\mathbf{(15000 + j\,0)}}\ \text{Watts}$$

For load C , $S_C = VI\cos\theta + j\,VI\sin\theta = VI\angle\theta$ where $\theta = \cos^{-1}pf$ is the impedance angle.

Thus for load C ; $S_C = 10000\angle\cos^{-1}0.8 = 10000\angle -36.87^\circ$ where the $-$ sign is used since a 'leading pf' is specified.

ie $S_C = 10000\angle -36.9^\circ = \underline{\mathbf{(7996.85 - j6004.2)}}$ Watts

(b) $S_{total} = S_A + S_B + S_C = 47996.8 + j\,23224.04 = 53320.25\angle 25.8^\circ$ Watts

Since $|S_{total}| = \sqrt{3}\,V_L I_L$, $I_L = 53320.25/\sqrt{3}(208) = \underline{\mathbf{148}}$ A

and the combined pf is $PF_{total} = \cos(\text{angle of } S_{total}) = \cos 25.8^\circ = \underline{\mathbf{0.9}\ \text{lagging}}$

1. For the network shown in Fig.1, using **mesh analysis**, with the indicated mesh currents,

(a) determine the power dissipated in the 10 Ω resistor,

and (b) determine the voltage v_s.

(8 marks)

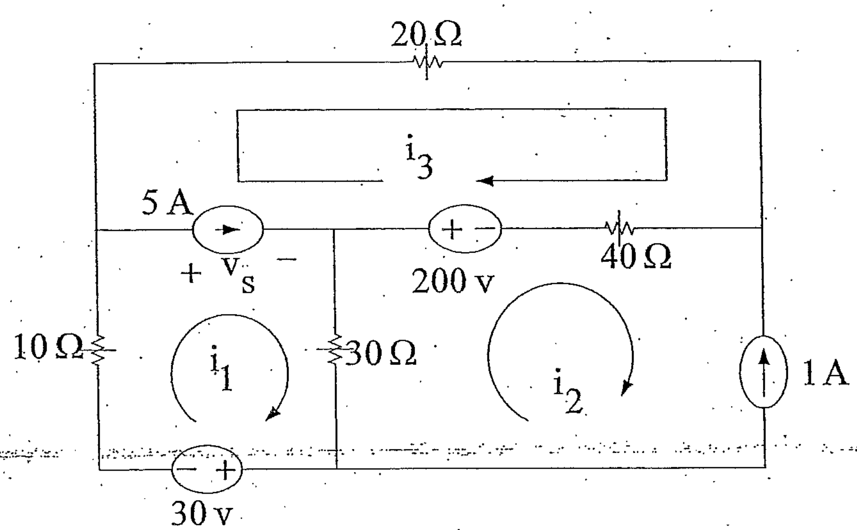


Fig.1

Solution:

Constraints are $i_1 - i_3 = 5$ and $i_2 = -1$

KVL around the path not containing current sources is :

$30 + 10i_1 + 20\,i_3 + 40\,(i_3 - i_2) - 200 + 30\,(i_1 - i_2) = 0$

Or $40\,i_1 + 60\,i_3 + (30 + 40 - 200 + 30) = 0$ or $4\,i_1 + 6\,i_3 = 10$

ie : $4\,i_1 + 6(i_1 - 5) = 10$ or $10i_1 = 40$

Therefore $i_1 = \mathbf{4\,A}$, $i_3 = 4 - 5 = \mathbf{-1\,A}$

(a) The power dissipated in the 10 Ω resistance is $= (i_1)^2\,(10) = 16(10) =$ **160 Watts**

(b) KVL : $10i_1 + V_s + 30\,(i_1 - i_2) + 30 = 0$ or $V_s = -30 - 30(4+1) - 40 =$ **-220 Volts**

#2 OMITTED (op amp)

(11)

3. In the circuit shown in Fig.3,
   (a) use Thevenin's theorem to determine the current i_{ab} which will flow through the switch after it is closed,
   (b) determine the power delivered by the Thevenin source under the condition of (a).

(6 marks)

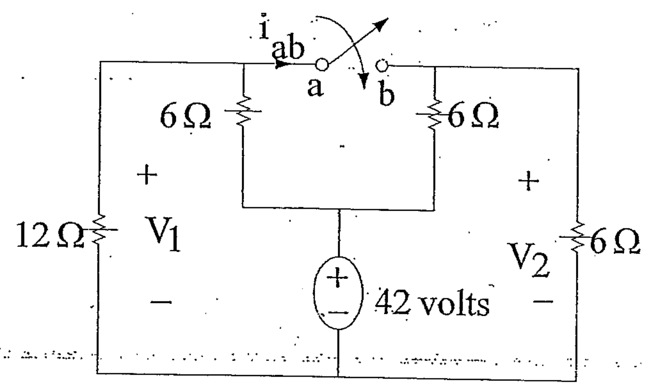


Fig.3

Solution: Removing the switch, we have

$$V_{Th} = V_{ab}(oc) = V_1 - V_2 = 42\,(12/18) - 42\,(6/12) = 28 - 21 = 7 \text{ volts}$$

The Thevenin resistance $R_{Th} = R_{ab}(dead) = (12P6) + (6P6) = 4 + 3 = 7\,\Omega$

(a) Therefore with the switch closed (after re-insertion), $i_{ab} = V_{Th}/R_{Th} = 7/7 = \underline{\mathbf{1}\ \text{A}}$

(c) The power delivered by the Thevenin source is (7)1 = <u>7 Watts</u>

4. The circuit shown in Fig.4 is in the steady state at $t = 0_-$ with the switch in position 'a'. The switch is moved to position 'b' at time $t = 0$. Determine $v(t)$ for $t > 0$.

(8 marks)

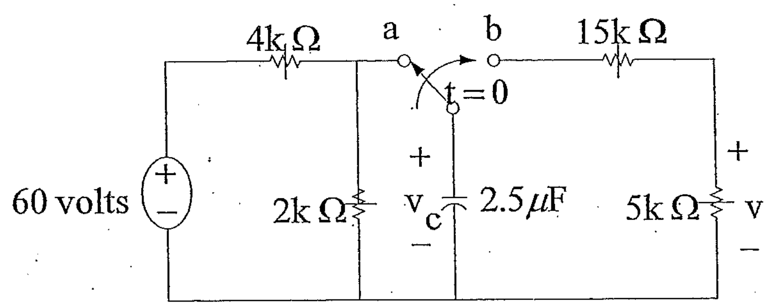


Fig.4

Solution: In the steady state at $t = 0-$, C is an oc and

$$V_c(0-) = V_c(0+) = 60\ (2/6) = 20 \text{ volts.}$$

For $t > 0$, $v = (5/20)V_c$ and KCL yields $(2.5)10^{-6}\, dV_c/dt + V_c/20000 = 0$

ie $0.05\, dV_c/dt + V_c = 0$

or $V_c(t) = A\, e^{-20t} + 0$ where $A = V_c(0) = 20$

Therefore $V_c(t) = 20e^{-20t}$ volts and $v(t) = \underline{\mathbf{5e^{-20t}}}$ volts

5. In the circuit of Fig.5, the initial conditions are known to be $i_L(0) = 666\ \mu A$ and $v_c(0) = 0$. Determine $i_R(t)$ for $t > 0$. The values of the various components are: $R = 60\ \Omega$, $C = 333\ \mu F$ and $L = 5.1$ H.

(8 marks)

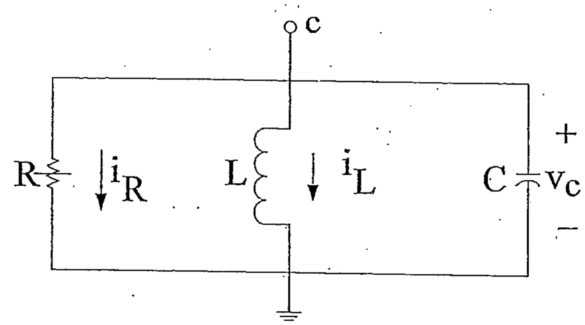


Fig.5

Solution: First note that $i_R(t) = V_c(t)/R = V_c(t)/60$

For a parallel RLC circuit, $\alpha = 1/2CR$ and $\omega_o = 1/\sqrt{LC}$:With the given values, $\alpha \approx 25.025$, $(\omega_o)^2 = 589$, $\omega_o \approx 24.27$ and $\alpha > \omega_o$ and the roots of the CE are

$$S_{1,2} = -25.025 \pm 6.1 = -18.925, -31.125$$

Since the circuit is source-less for t>0:

$$V_c(t) = A_1 e^{-18.925t} + A_2 e^{-31.125t} + 0$$

Now $\quad V_c(0) = 0 = A_1 + A_2$ and

$$dV_c(0)/dt = -18.925A_1 - 31.125A_2$$

But, $dV_c(0)/dt = i_c(0)/C$ and since $V_c(0) = 0$, $i_R(0) = 0$

and $i_c(0) = -i_L(0) = -0.666$ mA . Hence $dV_c(0)/dt = i_c(0)/C = -2$ volts/sec

ie We have : $\quad A_1 + A_2 = 0$ and $-18.925A_1 - 31.125A_2 = -2$

Solving $12.2A1 = -2$ or : $\quad A_1 = -A_2 \approx -0.164$

Hence $V_c(t) = 0.164\,[e^{-31.125t} - e^{-18.925t}]$

and $i_R(t) = V_c(t)/60 =$ **$2.73\,[e^{-31.125t} - e^{-18.925t}]$** mA

(14)

6. A variable-frequency sinusoidal source of voltage $v_g(t) = 212\cos(\omega t)$ volts and internal impedance $R_g = 40\ \Omega$ is connected to an RLC network as shown. It is required that the impedance 'seen' at the output terminals c-d of the network be 60 Ω resistive, that is, $Z_{cd} = (60 + j\,0)\ \Omega$.

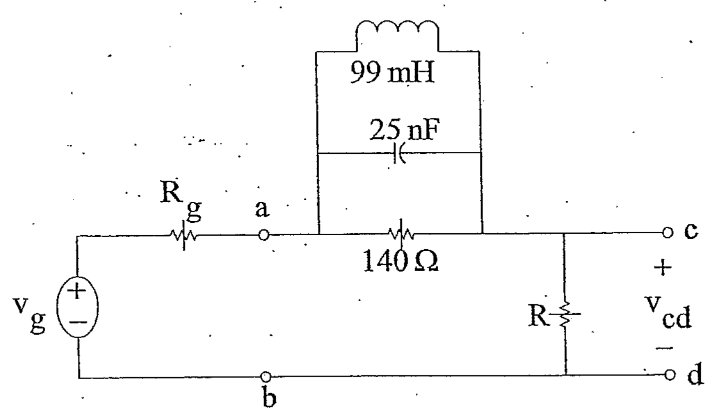


Fig.6

Determine

(a) the frequency $f_o = (\frac{\omega}{2\pi})$ at which the impedance Z_{cd} will be resistive,

(b) the required value of R which will result in $Z_{cd} = R_{cd} = 60\ \Omega$ at the frequency f_o,

(c) the voltage $v_{cd}(t)$ in the form $V\cos(\omega t + \theta)$, if the frequency of the source is 10 kHz and R = 80 Ω.

(11 marks)

Solution: (a) The impedance Z_{cd} will be resistive when the parallel RLC section of the network is in resonance, ie when the frequency $f_o = 1/[2\pi\sqrt{LC}]$

ie $f_o = 1/[6.28\,(4.975\text{x}10^{-5})] \approx$ <u>3.2 kHz</u>

(b) At 3200 Hz, $Z_{cd} = R_{cd} =$ R (parallel) (140 +40) = 180R/ (180+R)

If $R_{cd} = 60\ \Omega$, 180R=10800 + 60R or R = 10800/120 = **<u>90</u>** Ω

(c) At 10 kHz, $\omega = 62800$ rad/s and the total impedance in series with R is

$$Z = 40 + 1/[0.00714 + j(\omega C - 1/\omega L)] = 40 + 1/[0.00714 + j0.00141]$$
$$= 40 + 134.799 - j\,26.619 = 174.8 - j\,26.6$$

$V_{cd} = 212\,[80/(80 + 174.8 - j\,26.6)] = 212\,[80/(254.8 - j\,26.6)] == 66.2\ \angle 6°$
or $v_{cd}(t) =$ **<u>66.2 cos (62800t + 6°)</u>**

(15)

7. A 4:1 ideal single-phase transformer delivers 30 Amperes (rms) at a voltage of $V = 85\ \angle 0^o$ volts (rms) to an inductive load (R and L in series) as shown in Fig.7. At a frequency of 60 Hz, the load power factor is 0.79 (lagging). Determine

(a) the values of R and L,
(b) the impedance Z_p 'seen' at the primary terminals a-b,
(c) the complex power delivered.

(8 marks)

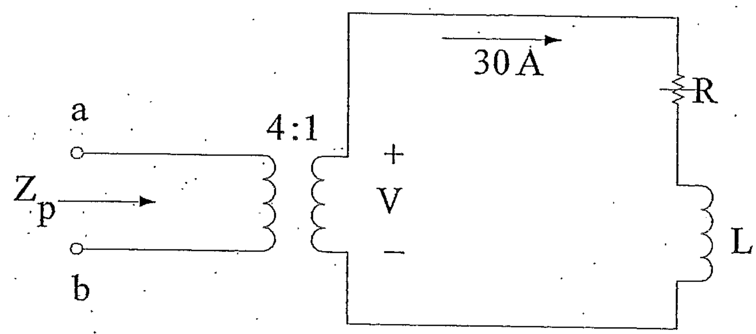


Fig.7

Solution: (a) $V_s = V = 85\angle 0^o$ and $I_s = 30\angle -\cos^{-1}(0.79)$,since the pf is lagging
ie $I_s = 30\angle -\cos^{-1}(0.79) = 30\angle -37.8^o$

Therefore , the impedance angle $\theta = \cos^{-1}(0.79) = 37.8^o$

Thus , the impedance Z_s on the secondary side is $= 85/30\angle 37.8^o$

or $Z_s = 2.833\angle 37.8^o = 2.24 + j\,1.74 = R + j\omega L$

Hence $R = \underline{2.24}\ \Omega$ and $L = 1.74/377 = \underline{4.62}$ mH

(b) $Z_p = n^2 Z_s = 16\ Z_s = \underline{\mathbf{45.33}}\angle \mathbf{37.8^o} = \underline{\mathbf{35.8 + j\,27.8}}$ ohms

(c) $S = V_s I_s^* = 85\angle 0^o(30\angle +37.8^o) = \underline{\mathbf{2550}}\ \angle \mathbf{+37.8^o} = \underline{\mathbf{2014.9 + j\,1562.9}}$ Watts

8. For the unbalanced 3-phase circuit shown in Fig.8, determine

(a) the current I_n in the neutral line;

(Hint: Use **Superposition** theorem)

(b) the total average power P and the total reactive power Q.

The various voltages are: $V_a = 110\angle 0^0$ volts, $V_b = 110\angle -120^0$ volts, and $V_a = 110\angle 120^0$ volts,

(10 marks)

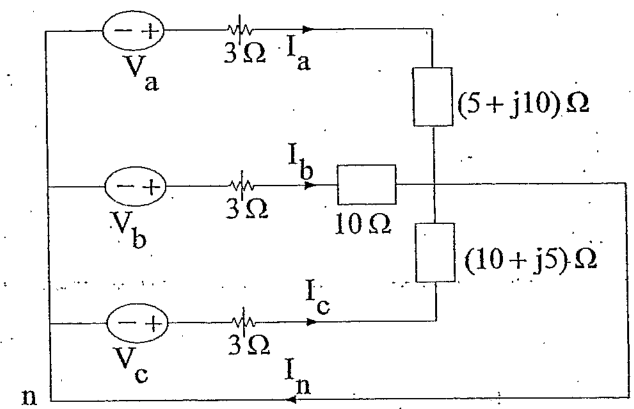


Fig.8

Solution: (a) The current in the neutral line is easily obtained by superposition noting that in each of the three sub-circuits the impedance associated with the 'killed' sources will be short-circuited by the neutral line !

Thus , the neutral current $I_n = I_a + I_b + I_c$

ie $$I_n = 110\angle 0^\circ/(3+5+j10) + 110\angle -120^\circ/(3+10) + 110\angle 120^\circ/(3+10+j5)$$
$$= 8.59\angle -51.3^\circ + 8.46\angle -120^\circ + 7.9\angle 99^\circ$$
$$= 5.37 - j\,6.7 - 4.23 - j7.33 - 1.24 + j7.8 = -0.1 + j\,6.23 = \underline{\mathbf{6.23}}\angle 91^\circ \text{ A}$$

(b) The total complex power $S = S_a + S_b + S_c$ where

$S_a = V_a I_a^* = 110\angle 0^\circ .\ 8.59\angle +51.3^\circ = 944.9\angle 51.3^\circ = 590.8 + j737.43$ watts

$S_b = V_b I_b^* = 110\angle -120^\circ .\ 8.46\angle +120^\circ = 930.6\angle 0^\circ = 930 + j\,0$ watts

$S_c = V_c I_c^* = 110\angle 120^\circ .\ 7.9\angle -99^\circ = 869\angle 21^\circ = 811.28 + j311.42$ watts

Summing $S = S_a + S_b + S_c = 2332.08 + j\,1048.85$ watts $= P + j\,Q$ watts

ie Total Average Power $P = 2332.08$ W $\approx$ **<u>2.33 kW</u>**

Total Reactive Power $Q = 1048.85$ W $\approx$ **<u>1.05 kVAR</u>**

1. For the network of Fig.1, determine the node voltages v_a, v_b, v_c and v_d, using **nodal analysis**. The various component values are:
$R_1 = 20$ ohms, $R_2 = 8$ ohms, $R_3 = 40$ ohms, $R_4 = 10$ ohms,
$v_{s1} = 15$ volts, $v_{s2} = 5.5$ volts and $i_s = 1.25$ amperes.

(10 marks)

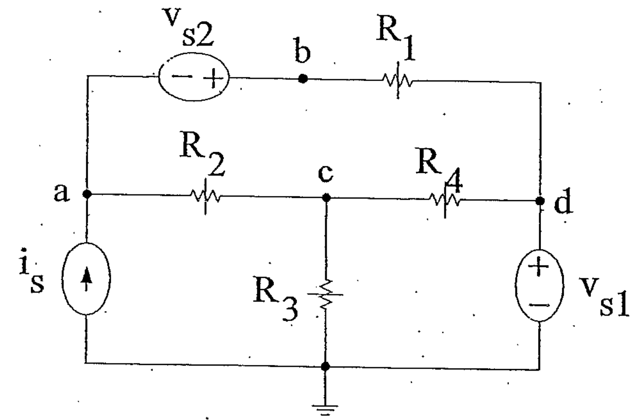


Fig.1

Solution: Node voltage constraints: $V_d = 15$, $V_b - V_a = 5.5$

The node equation for V_c is : $(V_c - V_a)/R_2 + V_c/R_3 + (V_c - V_d)/R_4 = 0$

ie $(V_c - V_a)/8 + V_c/40 + (V_c - 15)/10 = 0$

or $-5V_a + 10V_c = 60$

The supernode node equation for "V_a, V_b" is

$(V_a - V_c)/R_2 + (V_b - V_d)/R_1 = i_s$,

ie $(V_a - V_c)/8 + (V_b - 15)/20 = 1.25$

ie $(V_a - V_c)/8 + (5.5 + V_a - 15)/20 = 1.25$

or $7V_a - 5V_c = 69$

Solving $V_a = [(60)(-5)-(69)(10)]/[(-5)(-5)-(7)(10)] = -990/-45 =$ 22 volts

and $V_b = V_a + 5.5 =$ 27.5 volts

Solving $V_c = [(-5)(69)-(7)(60)] = -765/-45 =$ 17 volts

V_d is given, $V_d =$ 15 volts

#2 OMITTED (opamp)

(18)

3. In the network of Fig.3, the switch is open for a long period of time and is closed at $t = 0$. Determine $v_c(t)$ and $i_c(t)$ for $t > 0$. The various component values are: $R_1 = R_2 = R_3 = 15$k ohms, $i_s = 1$ mA, $v_s = 20$ volts, $C = 10\ \mu$F.

(10 marks)

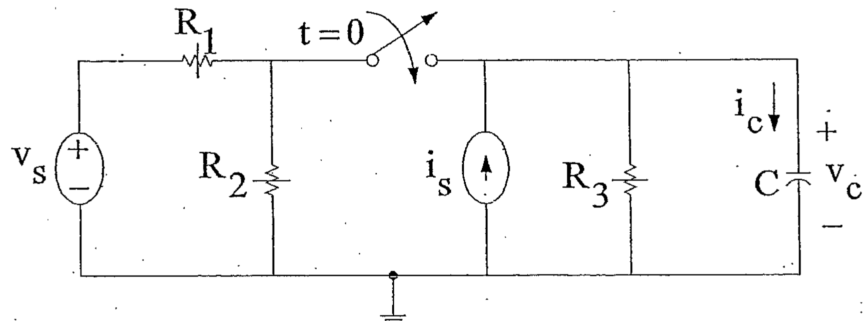


Fig.3.

Solution: At $t = 0-$, the switch was open and C was an oc (steadystate) and hence

$$V_c(0-) = V_c(0+) = i_s R_3 = 1\ (15) = 15 \text{ volts}$$

For $t > 0$, the circuit is an RC circuit with two sources

Nodal analysis (KCL in mA) yields

$$V_c/ 7500 + (V_c - 20)/ 15000 + 10(10)^{-6} (dV_c /dt) - 0.001 = 0$$

$$3V_c + 0.15((dV_c /dt) = 15 + 20 = 35$$

or $\quad 0.05 dV_c(t)/dt + V_c(t) = 35/3 = 11.667$

Hence $V_c(t) = A\, e^{-t/\tau} + 35/3$ and $A = V_c(0+) - 35/3 = 15 - 35/3 = 10/3$

$$V_c(t) = 35/3 + (10/3)\, e^{-20t} = \underline{[35 + 10\, e^{-20t}] /3} \text{ volts}$$

$$i_c(t) = C\, dV_c/dt = 10^{-5}\, (-200/3) e^{-20t} \text{ A} = \underline{(-2/3)\, e^{-20t}} \text{ A or } -\underline{0.6667\, e^{-20t}} \text{ mA}$$

4. Fig.4 shows a RLC network.
(a) Determine the Norton's equivalent current source to the left of the terminals a-d. (Hint: Use **Superposition Theorem**)
(b) Hence, obtain the Norton's equivalent circuit to the left of a-d.
(c) The switch is closed at $t = 0$. Determine $v_c(t)$ for $t > 0$, assuming zero initial conditions.
The various component values are: $R_1 = R_2 = R_3 = R_4 = 10$ ohms, $v_s = 20$ volts, $i_{s1} = 5$ A and $i_{s2} = 10$ A, C = 1 F, and L = 4 H.

(10 marks)

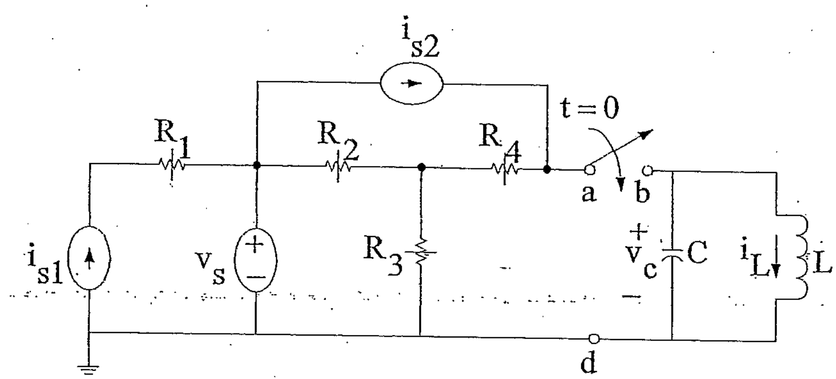
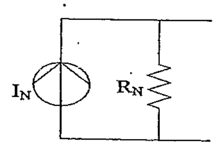


Fig.4

<u>Solution:</u> (a) $I_N = I_{ab}$ (SC)
By superposition, $I_N = 0 + \{V_s/[(R_2 + R_3R_4/(R_3+R_4)]\}[R_3/(R_3+R_4)] + i_{s2}$
ie $I_N = 0 + \{V_s/[(R_2 + R_3R_4/(R_3+R_4)]\}[R_3/(R_3+R_4)] + i_{s2}$
$I_N = 0 + \{20/15\}[10/20] + 10 = 10/15 + 10 = 160/15 = 32/3 =$ <u>10.667 A</u>
(b) $R_N = R_{ad}$(dead) $= R_4 + R_3$ P $R_2 = 10 + 5 =$ <u>15 Ω</u>



(c) For $t > 0$, the circuit is a Parallel RLC circuit connected to the current source I_N. $\alpha = 1/2RC = ½(15)(1) = 1/30$ and $\omega_n = 1/\sqrt{LC} = ½$. . $\alpha < \omega_n$ and the circuit is <u>underdamped</u>. $\omega_d = \sqrt{\omega_n^2 - \alpha^2} = 0.498 \approx 0.5$

(d)
$V_c(t) = [A_1 \cos 0.5\,t + A_2 \sin 0.5\,t\,]e^{-t/30} + 0$ since $V_c(\infty) = 0$ as $L \to$ sc .
At t=0+, $V_c(t) = 0$ and hence , $A_1 + 0 = 0$ or $A_1 = 0$
$dV_c(0)/dt = A_1(-1/30) + 0.5\,A_2 = (32/3)/1$, since , at t=0+, $dV_c(0)/dt = I_N/C$
Hence $A_2 = (32/3)/0.5 = 64/3 = 21.33$ and , $V_c(t) =$ <u>$(64/3)\,e^{-t/30}\sin 0.5\,t$</u>, volts

5. A RLC network is shown in Fig.5.

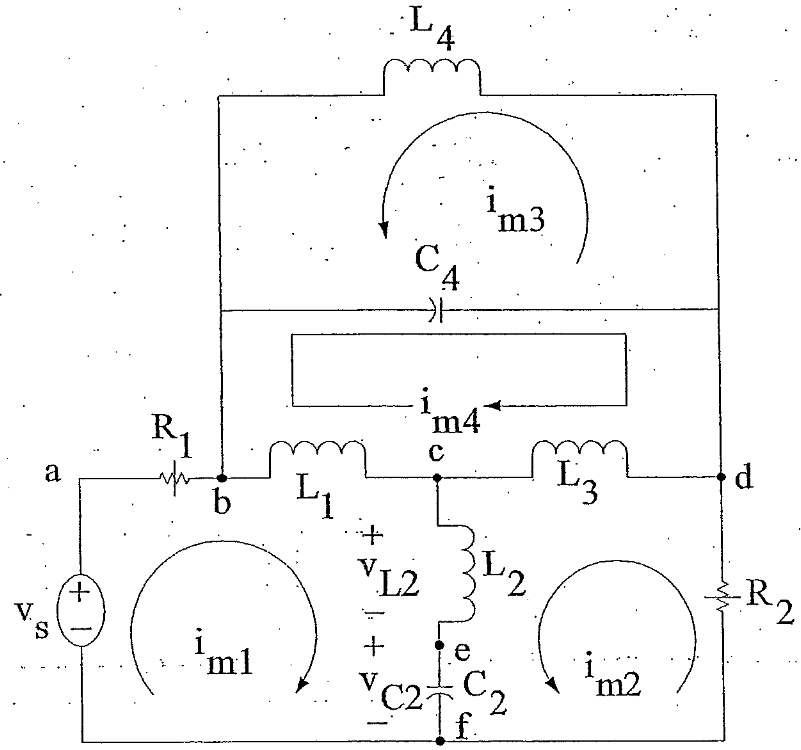


Fig.5.

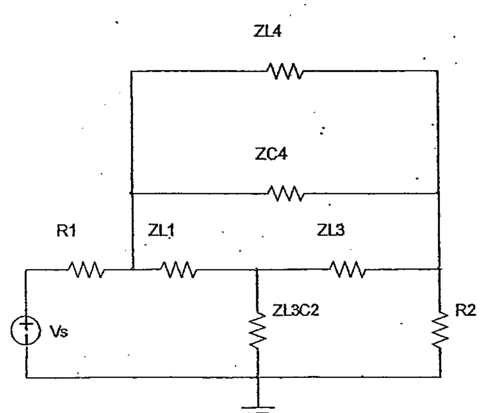
(a) Draw the equivalent circuit in the frequency domain.

(b) The various component values are given as: $v_s = 100 \cos(5000\,t)$ volts, $R_1 = R_2 = 5$ ohms, $L_1 = L_3 = 2$ mH, $L_2 = 1$ mH, $L_4 = 4$ mH, $C_2 = 40$ µF and $C_4 = 10$ µF.

Using the **meshes shown,** determine the voltages v_{L2} and v_{C2}, expressing them in the form $v = V_m \cos(\omega t + \theta_v)$ volts.

(10 marks)

Solution: (a) The equivalent circuit in the frequency domain is as below



$R_1 = R_2 = 5$ ohms,.Since and $\omega = 5000$ radians/sec, $Z_{L1} = Z_{L3} = j\omega\, 2\text{ mH} = j10\ \Omega$
.Similarly $Z_{L4} = j\,\omega\, 4\text{ mH} = j20\ \Omega$, $Z_{C4} = -j/\omega C_4 = -j20$,and
$Z_{L3C2} = j(\omega L_2 - 1/\omega C_2) = j\,(5\text{-}5) = 0$ and $V_s = 100\angle 0^\circ$

(c) The mesh equations are

$-100 + 5I_{m1} + j10[I_{m1} - I_{m4}] + (0)[I_{m1} + I_{m2}] = 0$ or $(5+j10)\, I_{m1} - j10I_{m4} = 100$

$5I_{m2} + j10[I_{m2} + I_{m4}] + (0)[I_{m1} + I_{m2}] = 0$ or $(5+j10)\, I_{m2} + j10I_{m4} = 0$

$-j20[I_{m3} + I_{m4}] + j20I_{m3} = 0$ or $j(20\text{-}20)\, I_{m3} - j20I_{m4} = 0$

$-j20[I_{m4} + I_{m3}] + j10[I_{m4} + I_{m2}] + j10\,[I_{m4} - I_{m1}] = 0$

ie $j(20\text{-}20)\, I_{m4} - j20I_{m3} + j10I_{m2} - j10\, I_{m1} = 0$ or $I_{m3} = -(0.5)\, I_{m1}$

From the third equation, $I_{m4} = 0$. With $I_{m4} = 0$, the second equation gives $I_{m2} = 0$.

The first equation gives $I_{m1} = 100/(5+j10) = 100/11.18\angle 63^\circ.4 = 8.94\angle -63^\circ.4$ A

and from the fourth equation $I_{m3} = -(0.5)\, I_{m1} = 4.47\angle 180^\circ - 63^\circ.4 = 4.47\ \angle 116^\circ.6$ A

The phasor voltages V_{L2} and V_{C2} are, effectively,

$V_{L2} = j\omega L_2\, I_{m1} = j5\,[8.94\angle -63^\circ.4] = 44.7\angle 90^\circ - 63^\circ.4 = 44.7\angle 26^\circ.6$ volts
and
$V_{C2} = [-j/\omega C_2\,]I_{m1} = -j5\,[8.94\angle -63^\circ.4] = 44.7\angle -90^\circ - 63^\circ.4 = 44.7\angle -153^\circ.4$ volts

The corresponding time-domain expressions are

$v_{L2}\,(t) =$ 44.7 cos [5000 t + 26°.6] volts

and

$v_{C2}\,(t) =$ 44.7 cos [5000 t −153°.4] volts

22

6. Fig.6 shows an impedance-matching circuit in which the RC network to the right of terminals a-b is matched to the source composed of v_s and R_s, such that maximum power is delivered to the RC-network. An ideal transformer of ratio 1 : n and the inductance L are used to obtain the matching conditions. The values in the circuit are: $R_1 = 402$ ohms, $R_2 = 10k$ ohms, $C = (22) \times 10^{-9}$ F, $v_s = 4.8$ v (rms), $R_s = 60$ ohms and the operating frequency is 1000 Hz.

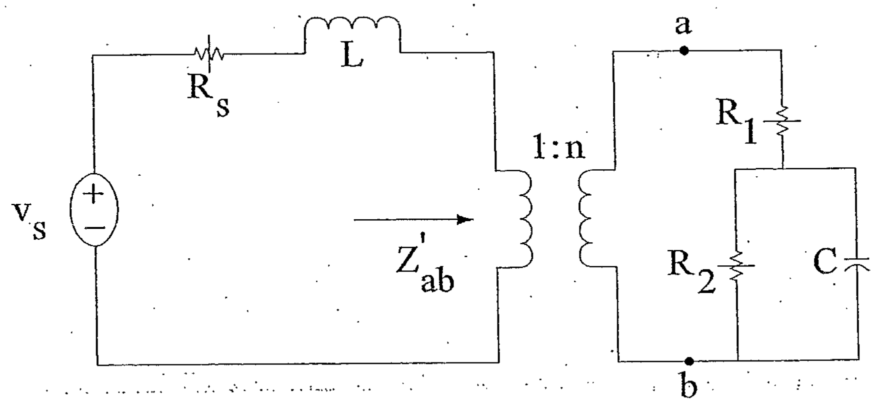


Fig.6

(a) Find the impedance Z_{ab} connected to the secondary of the transformer.
(b) Determine the values of n and L required in order to obtain the maximum power transfer.
(c) Determine the power P delivered under the 'matched' condition.

(12 marks)

Solution: (a) f = 1000 Hz corresponds to $\omega = 2\pi(1000) = 6280$ radians/sec

$$Z_{ab} = R_1 + (R_2/j\omega C)/[R_2 + 1/j\omega C] = 402 + [-j7.238(10)^7/(10^4 - j7237.98)]$$
$$= 402 + [\ 7.238(10)^7 \angle -90° / 12344.57 \angle 35°.9\ ]$$
$$= 402 + 5863.3 \angle -54°.1 = 402 + 3438 - j4749.5$$
$$= \underline{3840 - j4749.5} \text{ Ohms} = \underline{6107.65 \angle -51°} \text{ Ohms}$$

(b) Primary-reflected impedance $Z'_{ab} = Z_{ab}/n^2 = [\underline{3840 - j4749.5}]/n^2$

For maximum power transfer, we must have $\text{Re}\, Z'_{ab} = R_s = 60$ and $\text{Im}\, Z'_{ab} = -\omega L$

ie $3840/n^2 = 60$ or $n = (3840/60)^{0.5} = 8$ Hence $\underline{n = 8}$

Equating $\text{Im}\, Z'_{ab} = -4749.5/64 = -\omega L = -6280\, L$, we obtain $L = \underline{11.8 \text{ mH}}$

(c) $P = (V_s)^2/4R_s = (4.8)^2/240 = \underline{96 \text{ mW}}$

7. Fig.7 shows a 3-phase source operating at 60 Hz connected to a Y-connected load impedance $Z = R + j\omega L$ through lines of impedance $Z_\ell = (0.2 + j\,0.6)$ ohms. The resistive component of the load R = 5 ohms and the load power factor is 0.64 lagging. The other components are: $V_a = 208\angle 20^\circ$ volts, $V_b = 208\angle -100^\circ$ volts and $V_c = 208\angle -220^\circ$ volts (all the voltages being in rms).

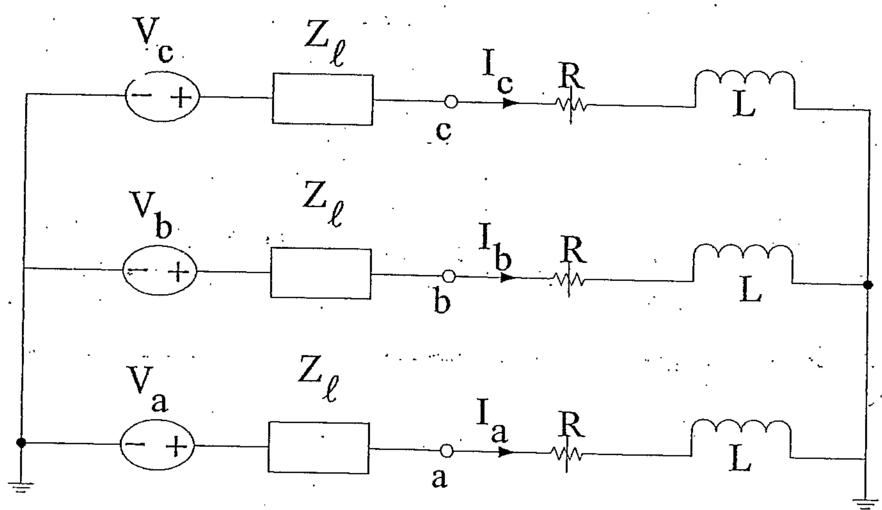


Fig.7

(a) Determine the value of the inductance L in the load.

(b) Determine the line-current I and hence the total real and reactive powers P_L and Q_L respectively in the load.

(8 marks)

Solution: (a) The load pf is 0.64 lagging, hence the load impedance angle

$$\theta = \cos^{-1}(0.64) = \angle 50^\circ.2 = \tan^{-1}(\omega L/R)$$

Therefore $(\omega L/R) = \tan\angle 50^\circ.2 = 1.2$ and

$$L = 1.2\,R/\omega = (1.2)(5)/377 = \underline{15.9\text{ mH } (\approx 16\text{mH})}$$

(b) The system is balanced. Each phase of the load impedance is $Z_L = 5 + j\,(377)(0.016) = (5 + j6)\ \Omega$ and each total impedance seen by each source is $Z_{total} = (5.2 + j6.6)\ \Omega$

Eg The line current $I_a = V_a/Z_{total} = 208\angle 20^\circ / (5.2+j6.6) = 208\angle 20^\circ / 8.4\angle 51^\circ.77$

$$\approx 24.76\angle -32^\circ$$

The total average power <u>in the load</u>, $P = 3\,I^2\,\text{Re}Z$

$$= 3\,(24.76)^2(5) = 9195.8\text{ W or } \approx \underline{9.2\text{ kW}}$$

The total reactive power <u>in the load</u>, $Q = 3\,I^2\,\text{Im}Z = 3\,(24.76)^2(6)$

$$= 11035\text{ var or } \approx \underline{11.03\text{ kvar}}$$