COMP 335 Theoretical Computer Science
Winter 1997 Mid-term 1

.07

Time: 1 hour 15 minutes

Notes: (1) This exam will be graded out of 15 points.
(2) Only complete explanations will get full credit.

1. [2] State the pumping lemma for regular languages.

2. [3] Prove that the language $L_1 = \{a^i b^j a^j \mid i, j \geq 0\}$ is not regular.

3. [5] Prove that the following languages are regular:

   (a) $L_2 = \{w \in (a+b)^* \mid n_a(w)$ is odd or $n_b(w)$ is even $\}$

   (b) $L_3 = \{w \in (a+b)^* \mid |w| \geq 1$ and the last letter of w is the same as the first letter of $w\}$

4. Consider the language $L = \{w \in (a+b)^* \mid$ the string aba does not appear in $w\}$.

   (a) [2] Find a DFA that accepts L.

   (b) [1] Convert it into an equivalent right regular grammar.

   (c) [2] Find a regular expression that represents $\overline{L}$.

Bonus:

5. [1] Let $w = w_1 w_2 \ldots w_n$ be a string in Σ^* where each w_i is a symbol in the alphabet Σ. Then $w^R = w_n w_{n-1} \ldots w_2 w_1$. Given a regular expression for a language L, show how to construct a regular expression for $Reverse(L) = \{w \mid w^R \in L\}$.

Sample Sol^n

COMP 335 Theoretical Computer Science
Winter 1996 Mid-term 2

Time: 1 hour 15 minutes

1. [6] For each of the following languages, say whether or not it is context-free. Explain your answer.

   (a) $L_1 = \{uavb \mid u, v \in (a+b)^*, |u| = |v|\}$

   (b) $L_2 = \{a^n b^n c^k \mid n \leq k \leq 2n\}$

2. [3] Say true or false, providing a complete explanation for your answer. Answers without correct explanations will get NO credit.

   (a) If $L_1 \subseteq L_2$ and L_1 is not context-free, then L_2 is not a context-free language.

   (b) The grammar given below is unambiguous.
   $S \longrightarrow SS \mid aSb \mid bSa \mid \lambda$

   (c) The language $\{a^n b^{2n} \mid n \geq 0\}$ is a deterministic context-free language.

3. [4] Consider the language $L = \{a^l b^n c^p d^q \mid l + n = p + q\}$

   (a) Give a CFG that generates the language.

   (b) Give a push down automaton that accepts the same language. You may either give a direct construction, or convert the context-free grammar you derived above to an equivalent PDA.

4. [2] Suppose L is a context-free language and R is a regular language. Is $L - R$ necessarily context-free? How about $R - L$? Provide explanations for your answers.

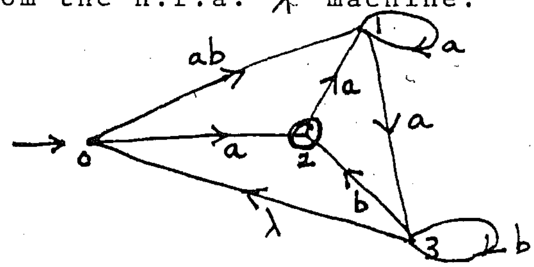
Bonus Question:

[2] Show that the conversion to Chomsky Normal Form can square the number of productions in a grammar.

COMP335/2 Section U Midterm Oct. 26th. 1993 Time: 30 minutes

No notes, no books. Give full explanations for full marks.

1. From the n.f.a.-λ machine:



Construct an equivalent deterministic finite automaton. Give all steps in your construction.

2. (a) By drawing a graph of dependency among the non-terminals, or otherwise, show that the grammar given generates a finite language, and give a derivation of a longest word.

$S \rightarrow XY, \quad X \rightarrow YZ \mid a, \quad Y \rightarrow ZZ \mid b, \quad Z \rightarrow a$

(b) Adding the production rule $Z \rightarrow XY$, show that the language is now infinite & give a regular expression for an infinite string in it.

3. Construct a p.d.a. from the grammar:

$S \rightarrow zXYz, \quad X \rightarrow aXa \mid z, \quad Y \rightarrow bYb \mid z$

.... Name

.... Student Id

Concordia University
Department of Computer Science

COMP 335 – Introduction to Theoretical Computer Science

Section WW, Fall 1998

Mid-Term Test
November 24, 1998

The time allowed is 90 minutes. Total 20 marks. No materials are allowed.

1. Let $L = \{x \in \{a,b\}^* \mid n_a(x) < 3n_b(x)\}$. Show that L is nonregular.

   SOLUTION. Let $x = a^{3n-1}b^n$. Since $|uv| \le n$, $uv = a^k$ for some k, $0 < k \le n$. Since $|v| > 0$, $v = a^j$ for some j, $0 < j \le k$. Hence $n_a(uv^2w) = n_a(a^{3n-1+j}b^n) = 3n - 1 + j \ge 3n = 3n_b(a^{3n-1+j}b^n)$, so $uv^2w \notin L$.

2. Find a CFG generating the language $\{x \in \{a,b\}^* |$the two middle symbols of x are equal$\}$.

   SOLUTION. $S \rightarrow aa \mid bb \mid aSa \mid aSb \mid bSa \mid bSb$.

3. Find a CFG in Chomsky normal form generating the same language as the following grammar.

$$
\begin{array}{lcl}
S & \rightarrow & SABC \mid ab \\
A & \rightarrow & Aab \mid Bb \\
B & \rightarrow & CDc \mid b \mid \Lambda \\
C & \rightarrow & BD \mid abD \\
D & \rightarrow & Dc \mid \Lambda
\end{array}
$$

SOLUTION.

1. Eliminating Λ-productions.

$$
\begin{array}{lcl}
S & \rightarrow & SABC \mid SAB \mid SAC \mid SA \mid ab \\
A & \rightarrow & Aab \mid Bb \mid b \\
B & \rightarrow & CDc \mid Cc \mid Dc \mid c \mid b \\
C & \rightarrow & BD \mid B \mid D \mid abD \mid ab \\
D & \rightarrow & Dc \mid c
\end{array}
$$

2. Eliminating unit-productions.

$$
\begin{array}{lcl}
S & \rightarrow & SABC \mid SAB \mid SAC \mid SA \mid ab \\
A & \rightarrow & Aab \mid Bb \mid b \\
B & \rightarrow & CDc \mid Cc \mid Dc \mid c \mid b \\
C & \rightarrow & BD \mid CDc \mid Cc \mid Dc \mid c \mid b \mid abD \mid ab \\
D & \rightarrow & Dc \mid c
\end{array}
$$

3.

$$
\begin{array}{lcl}
S & \rightarrow & SABC \mid SAB \mid SAC \mid SA \mid EF \\
A & \rightarrow & AEF \mid BF \mid b \\
B & \rightarrow & CDG \mid CG \mid DG \mid c \mid b \\
C & \rightarrow & BD \mid CDG \mid CG \mid DG \mid c \mid b \mid EFD \mid EF \\
D & \rightarrow & DG \mid c \\
E & \rightarrow & a \\
F & \rightarrow & b \\
G & \rightarrow & c
\end{array}
$$

4.

$$
\begin{array}{lcl}
S & \rightarrow & SY_1 \mid SY_3 \mid SY_4 \mid SA \mid EF \\
A & \rightarrow & AY_5 \mid BF \mid b \\
B & \rightarrow & CY_6 \mid CG \mid DG \mid c \mid b \\
C & \rightarrow & BD \mid CY_6 \mid CG \mid DG \mid c \mid b \mid EY_7 \mid EF \\
D & \rightarrow & DG \mid c \\
E & \rightarrow & a \\
F & \rightarrow & b \\
G & \rightarrow & c \\
Y_1 & \rightarrow & AY_2 \\
Y_2 & \rightarrow & BC \\
Y_3 & \rightarrow & AB \\
Y_4 & \rightarrow & AC \\
Y_5 & \rightarrow & EF \\
Y_6 & \rightarrow & DG \\
Y_7 & \rightarrow & FD
\end{array}
$$

4. Give a deterministic PDA recognizing the language generated by the following grammar.

$$\begin{aligned} S &\rightarrow S_1\$ \\ S_1 &\rightarrow aAbB \mid bB \\ A &\rightarrow bA \mid \Lambda \\ B &\rightarrow aB \mid b \end{aligned}$$

SOLUTION. Top-down parser:

State	Input	Stack symbol	Move(s)
q_0	Λ	Z_0	(q_1, SZ_0)
q_1	Λ	S	$(q_1, S_1\$)$
q_1	a	S_1	$(q_a, aAbB)$
q_1	b	S_1	(q_b, bB)
q_1	a	A	(q_a, Λ)
q_1	b	A	(q_b, bA)
q_1	$\$$	A	$(q_\$, \Lambda)$
q_1	a	B	(q_a, aB)
q_1	b	B	(q_b, b)
q_1	a	a	(q_1, Λ)
q_1	b	b	(q_1, Λ)
q_1	$\$$	$\$$	(q_1, Λ)
q_a	Λ	a	(q_1, Λ)
q_b	Λ	b	(q_1, Λ)
$q_\$$	Λ	$\$$	(q_1, Λ)
q_1	Λ	Z_0	(q_2, Z_0)

CONCORDIA UNIVERSITY DEPARTMENT OF COMPUTER SCIENCE
COMP335 – THEORETICAL COMPUTER SCIENCE SUMMER 2000(CA)

Midterm Examination 1

1. *(20%)* The reverse of a string can be defined by the recursive rules

   $$a^R = a,$$

   $$(wa)^R = aw^R,$$

   for all $a \in \Sigma$, $w \in \Sigma^*$. Use this to prove that

   $$(uv)^R = v^R u^R,$$

   for all $u, v \in \Sigma^+$.

2. *(40%)* Let $L = L[(a+b)^*] - L[(bab)^*]$.
   i) Give a dfa that accepts L
   ii) Give a regular expression for L.

3. *(20%)* Let

   $$M_1 = (Q, \Sigma, \delta_1, q_0, F_1)$$

   $$M_2 = (P, \Sigma, \delta_2, p_0, F_2)$$

   be nfa's, with $Q \cap P = \Phi$, construct an nfa M such that

   $$L(M) = L(M_1) \cup L(M_2).$$

   Prove the construction.

4. *(20%)* Show that the language

   $$L = \{waw : w \in \{a, b\}^*\}$$

   is not regular.